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Volume of Hill Estimation for Engineering Earth Work

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Abstract
This study centralizes on the development of volume estimation calculation platform for earth engineering work. Estimation and comparison were done by using Foresight Civil Design and Survey (CDS) and Mathematica 7.0 software. Currently, Land Surveying companies are using Foresight Civil Design and Survey (CDS) software for its speed and automation. There are two methods used in Mathematica 7.0, which are Rational Quadratic Bezier Curve and Rational Cubic Bezier Curve. From this research, Rational Cubic Bezier Curve show its total hill measurement (volume) is nearer to value using Foresight Civil Design and Survey (CDS) software. Therefore, the Rational Cubic Bezier Curve is an alternative method to estimate the volume of the hill. The discrepancy between Rational Cubic Bezier Curve and Foresight Civil Design and Survey (CDS) mainly caused by curve tolerance has been generalized in Rational Cubic Bezier Curve.

Keywords: *volume estimation, Rational Quadratic Bezier Curve, Rational Cubic Bezier Curve*

Introduction

Volume calculation is crucial for cut and fill work. For civil engineering, volume calculations are important in order to identify the need of filling up the area with the cut volume (Yunos, 1996). Along with technology advancement, land survey companies have been using professional software to calculate soil volume. This research uses Foresight Civil Design and Survey (CDS) and Mathematica 7.0 software. Mathematica 7.0 has been used to estimate hill volume by using Rational Quadratic Bezier Curve and Rational Cubic Bezier Curve. The results were later compared to volume estimation by CDS software.

Bezier Curve is commonly used in computer graphic and geometry modelling. This algorithm uses 3D point's geometry parameters to generate a curve (Kenneth, 2000). Rational Bezier Curve is a Bezier Curve in 3D with rational polynomial or ratio of two polynomials.

This study uses both Rational Quadratic Bezier Curve and Rational Cubic Bezier Curve. Aiming to provide alternative software for cut and fill project for a small land survey company, this study utilizes Mathematica 7.0 to estimate hill's volume.
Consecutively, this research focuses on control value, h_i derivations and analysis for a smooth surface, estimates hill volume and analytical comparison for volume estimation between CDS and Mathematica 7.0. There are numerous researches for land surface volume calculation inclusive of soil, hill, mountain, coal mine among others.

Chun-Shung Chen and Hung-Cheng Lin (1990) in "Estimating Pit-Excavation Volume Using Cubic Spline Volume Formula" has extended cubic spline polynomial and develop a new formula to estimate coal mine excavation. Various size rectangles have been constructed by connecting two lines from ground profile point's selections. Irregular grid areas were later used to estimate coal mine excavation volume. Newly develop volume estimation formula in this research is accurate because it produces a smooth curve from cubic spline polynomial.
Pyramid Frustum Formula for Computing Volumes at Roadway Transition Areas" (Said, 1991) used Average-End-Area (AEA) and Pyramid Frustum (PF) formula to calculate cut and fill volume in road transition design. In AEA, volume was obtained by a multipliedaverage of the end areas by the distance between cross sections. In PF, the volume is calculated by multiplied base area by height and is divided by three. This research indicates a large error in AEA compared to PF. Therefore, PF is more suitable for road transition design volume calculations.

Methodology
This research uses simulation coordinate (x_i, y_i, z_i) data with i = 1,..., 10 as a control point. x_i, y_i are Easting and Northing, with z_i as a height. Assuming the base and top of the hill have an equal width, there will only be two Northing (y_1 and y_2). Control points i = 1-5 are coordinated for front surface and i = 6-10 are back surface. Curve for both surfaces are similar (Figure 1).

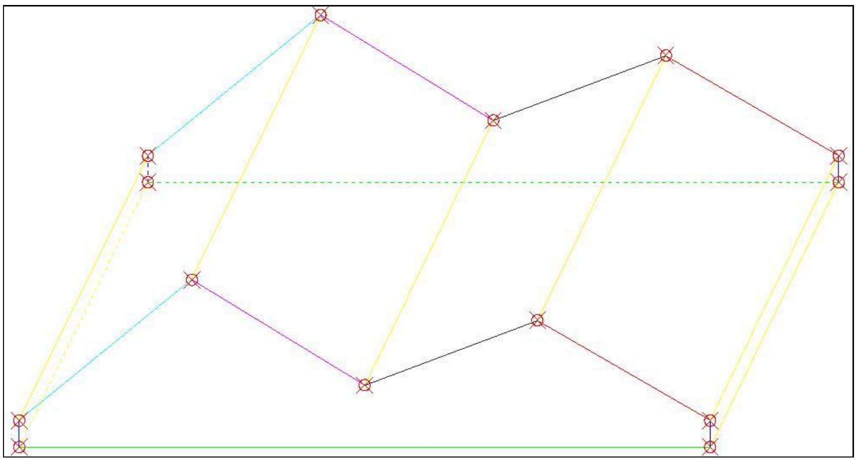


Figure 1: Hills front and back control points

This study uses both Rational Quadratic Bezier Curve and Rational Cubic Bezier Curve. Hill volume was determined by these steps:

Develop new equations with a constant Mean Sea Level (MSL) datum using Rational Quadratic Bezier Curve, equation (2) and Rational Cubic Bezier Curve, equation (4)

$$R_i(x) = r_i(x) - 7.5 \quad (1)$$

where $r_i(x)$ is a Rational Quadratic Bezier Curve or Rational Cubic Bezier Curve and 7.5 is an arbitrary value for Mean Sea Level.

$$r_i(t) = \frac{(1-t^2)z_i + 2(1-t)tw_i h_i + t^2 z_{i+1}}{(1-t)^2 + 2(1-t)tw_i + t^2} \quad (2)$$

where,

$$t = \frac{(x - x_i)}{(x_{i+1} - x_i)} \quad (3)$$

with control point r_i, $i = 1,\ldots,5$.

$$r_i(t) = \frac{(1-t^2)z_i + 2(1-t)^2 tw_i h_i + 2(1-t)t^2 w_{i+1} h_{i+1} + t^2 z_{i+1}}{(1-t)^2 + 2(1-t)^2 tw_i + 2(1-t)t^2 w_{i+1} + t^2} \quad (4)$$

where,

$$t = \frac{(x - x_i)}{(x_{i+1} - x_i)} \quad (5)$$

with control point r_i, $i = 1,\ldots,5$.

i) Calculate areas for each surface (A_1, A_2, A_3 & A_4)

$$A_i = \int_{x_{i-1}}^{x_i} R_i(x)dx \quad (6)$$

where $i = 1, 2, 3$ and 4.

ii) Calculate volume for each surface (V_1, V_2, V_3 & V_4)

$$V_i = A_i[y_2 - y_1] \quad (7)$$

where $i = 1, 2, 3$ and 4 and $y_2 - y_1$ is hill's width.

iii) Calculate hill's volume, V

$$V = V_1 + V_{21} + V_3 + V_4 \quad (8)$$

Both results were then compared to CDS result in order to evaluate the efficiency of the equations in calculating hill's volume.

Results and Discussion

The result from this research is obtained from Rational Quadratic Bezier Curve and Rational Cubic Bezier Curve. Both Bezier Curves were based on control value, h_i,weightage, w, Iterative Formula, h_{i+1}, hill's curvature based on weight, w and hill's volume.

The relationship between control value, h_i and weightage, w.

Control point, h_i, in this section is represented by h_1, the first control point. Table 1 are three arbitrary coordinates used in the simulations. Table 2 shows result for control point, h_i byRational Quadratic Bezier Curve with a different weightage, w. Figure 2 illustrates first and second curves from Rational Quadratic Bezier Curve. Rational Cubic Bezier Curve however, indicate a correlation between h_i and z_i. It was demonstrated by an Iterative Formula, $h_{i+1} = z_{i+1}$.

$$r_1 = \frac{\left[\left((1-t_1)^2 z_1\right) + \left(2(1-t_1)t_1 w h_1\right) + \left(t_1{}^2 z_2\right)\right]}{\left((1-t_1)^2\right) + \left(2(1-t)_1 t_1 w\right) + \left(t_1{}^2\right)} \tag{9}$$

$$r_2 = \frac{\left[\left((1-t_2)^2 z_2\right) + \left(2(1-t_2)t_2 w h_1\right) + \left(t_2{}^2 z_3\right)\right]}{\left((1-t_2)^2\right) + \left(2(1-t_2)t_2 w\right) + \left(t_2{}^2\right)}$$

(10)

Table 1: Coordinates (1-3)

i	x_i	y_i	z_i
1	3000.349	172.972	8.5
2	3667.444	172.972	18.5
3	4334.539	172.972	12.5

Table 2 : Value for h_1 by different w

w	h_1
0.8	31.293
0.9	19.185
1.0	29.440

Iterative Formula, h_{1+1}

Figure 2, 3 and 4 are the curves generated from Rational Quadratic Bezier Curve represented by first and second curves as (r_1 & r_2), second and third curves as (r_2 & r_3) and third and fourth curves as (r_3 & r_4). Figure 2 with r_1 and r_2, when $r_1'[x_2 = 3667.444] = r_2'[x_2 = 3667.444]$ and $x = x_2$, therefore $h_2 = \dfrac{h_1(x_3 - x_2) + (x_1 - x_3)z_2}{x_1 - x_2}$.

In Figure 3, both curves r_2 and r_3, when $r_2'[x_3 = 4334.539] = r_3'[x_3 = 4334.539]$ with $x = x_3$, it produced $h_3 = \dfrac{h_2(x_4 - x_3) + (x_2 - x_4)z_3}{x_2 - x_3}$.

For Figure 4, r_3 and r_4 curves, when $r_3'[x_4 = 5001.634] = r_4'[x_4 = 5001.634]$and $x = x_4$, resulted with $h_4 = \dfrac{h_3(x_5 - x_4) + (x_3 - x_5)z_4}{x_3 - x_4}$.

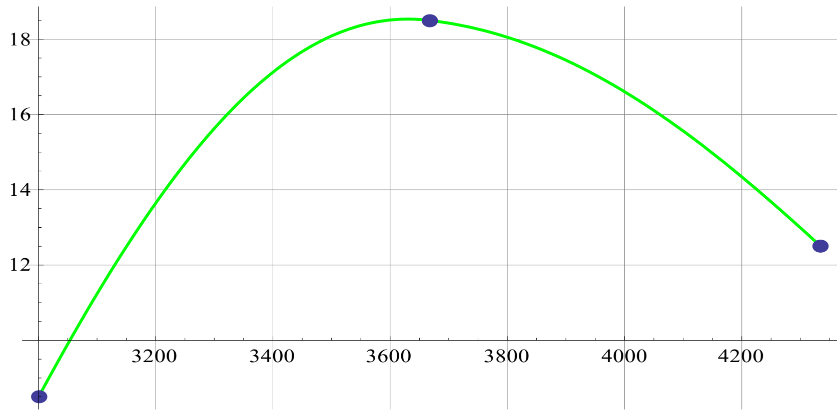


Figure 2: First and second curves by Rational Quadratic Bezier Curve

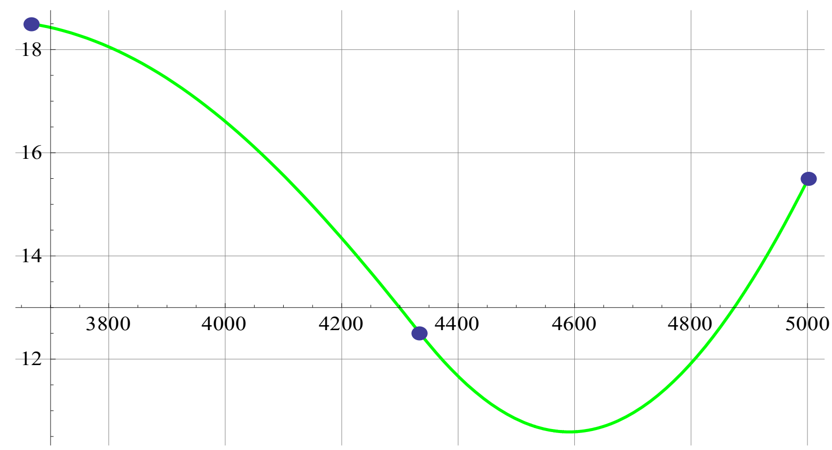


Figure 3: Second and third curves by Rational Quadratic Bezier Curve

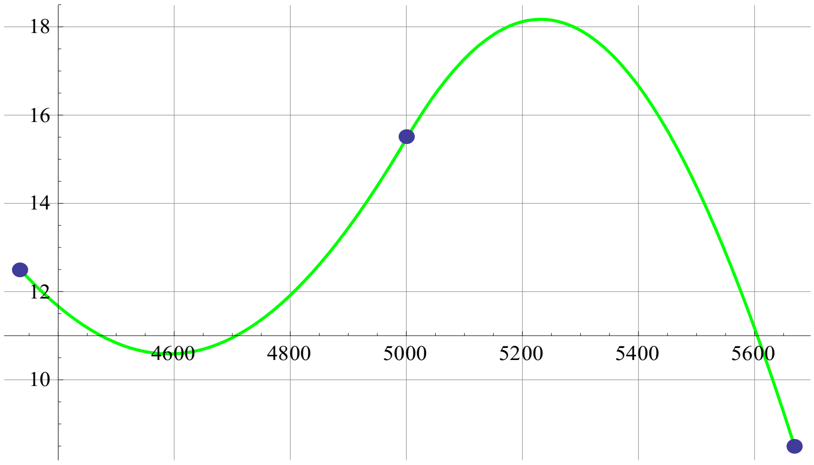


Figure 4: Third and fourth curves by Rational Quadratic Bezier Curve

In Rational Cubic Bezier Curve, the coordinate's differences have been set up to zero to represent C^1 continuity. For first and second curves $(r_1 \& r_2)$, when $r_1'[x_2 = 3667.444] = r_2'[x_2 = 3667.444]$ and $x = x_2$. For the second and third curves $(r_2 \& r_3)$, when $r_2'[x_3 = 4334.539] = r_3'[x_3 = 4334.539]$ and $x = x_3$. Finally, the third and fourth curves $(r_3 \& r_4)$, when $r_3'[x_4 = 5001.634] = r_4'[x_4 = 5001.634]$ with $x = x_4$.

Overall results for both formula are $h_2 = z_2$, $h_3 = z_3$ and $h_4 = z_4$. Therefore, equation (11) and (12) define the Iterative Formula for Rational Quadratic Bezier Curve and Rational Cubic Bezier Curve.

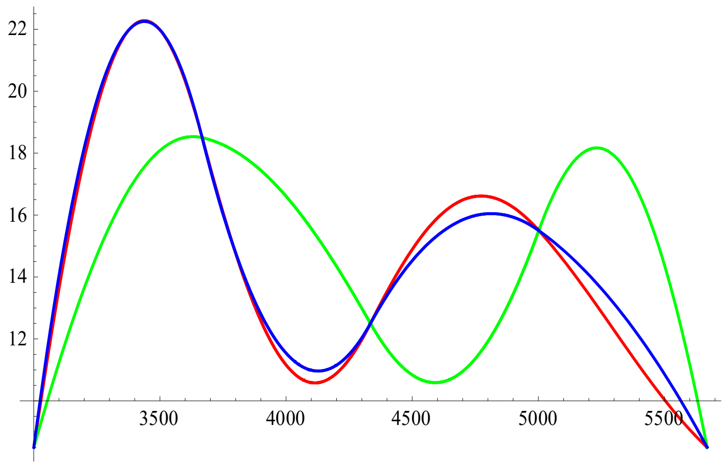
$$h_{i+1} = \frac{(x_i - x_{i+2})z_{i+1} - (x_{i+1} - x_{i+2})h_i}{x_i - x_{i+1}}$$

(11)

$$h_{i+1} = z_{i+1}$$

(12)

Curve's transition proxy of Weightage, w value



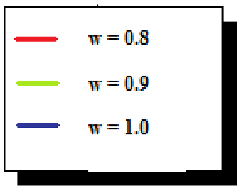


Figure 5: Rational Quadratic Bezier Curve with w variant

Figure 5 demonstrates the effect of weightage, w variant to the hill's curve shape in Rational Quadratic Bezier Curve.

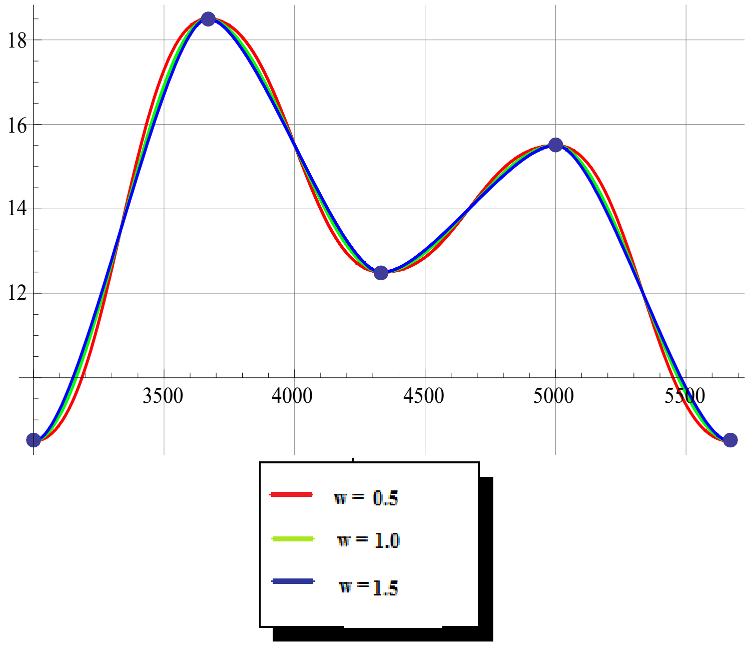


Figure 6: Rational Cubic Bezier Curve with w variant

Figure 6 displays the effect of weightage, w variant to the hill's curve shape in Rational Cubic Bezier Curve.

Hill's Volume

Table 3: Hill's volume estimation

Method				Volume ($unit^3$)
CDS				16677381.667
	Weightage, w	**Rational Bezier quadratic**	**Weightage,** w	**Rational Bezier cubic**
Mathematica				
	0.8	19192390.570	0.5	16677379.50
	0.9	19387790.787	1.0	16677379.50
	1.0	19568124.373	1.5	16677379.50

In regards to hill's volume calculations in Table 3, Rational Cubic Bezier Curve formula is almost identical to hill's volume computed by CDS compared to Rational Quadratic Bezier Curve formula. This insignificant volume difference however, was the succession of Rational

Cubic Bezier Curve tolerance. Hence Rational Cubic Bezier Curve is an adequate CDS alternate method to calculate hill's volume.

Conclusion

To conclude, it is safe to mention this research has delivered the objective. The h_i and weightage, w relationship only happened in Rational Quadratic Bezier Curve. For Rational Cubic Bezier Curve, the correlations mainly rely on z_i coordinates. The contrast in iterative formula for both Rational Quadratic Bezier Curve and Rational Cubic Bezier Curve formula were shown in equation (11) and (12) respectively. Both formulas however, have a different hill's curve for different weightage, w. Hill's volume uses Rational Cubic Bezier Curve, in which is overall nearer to CDS hill's volume. The slight difference is caused only by curve tolerance. This finding in another way offer cost effective options for land survey companies to be able to monitor cut and fill work progress. Progressing forward, it is suggested that future research should include higher detail in hill's shape and surface to gain better volume calculation precisions.

Suggestion

To summarize, Rational Cubic Bezier Curve method is able to resolve software and cost constraint to calculate cut and fill volume among land survey companies in Malaysia. However, the real contribution only can be verified by future research using scattered, inconsistent ground truth coordinates.

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