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Utilizing Trigonometric Bézier Curves for Reconstructing Arabic Calligraphy: Interpolating Quasi-Quartic and Quasi-Quintic Curves

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ABSTRACT

As technology continues to advance, Computer Aided Geometric Design (CAGD) is gaining popularity as a mathematical method for generating curves and surfaces. CAGD, a branch of applied mathematics, focuses on developing algorithms for creating smooth curves and surfaces efficiently. This paper explores the application of CAGD techniques to Khat Thuluth, a form of Arabic calligraphy known for its complexity and the skill required to create it. The study employs two methods of Trigonometric Bézier Curves, namely Quasi-Quartic and Quasi-Quintic, to reconstruct Arabic calligraphy. By examining how variations in shape parameters affect curve modifications, the research investigates the factors influencing the outcome. Comparison between the resulting figures and the original images, as well as the computational performance in terms of CPU time required for the entire calligraphy creation process, is conducted to evaluate the effectiveness of the two interpolation methods. The findings indicate that the Quasi-Quartic Trigonometric Bézier curve offers the most efficient reconstruction of Arabic calligraphy outlines, with a minimal CPU time of 8.453 seconds.

1. INTRODUCTION

Advancements in technology have popularized the mathematical method known as Computer Aided Geometric Design (CAGD) for generating curves and surfaces. CAGD, a field of applied mathematics, focuses on algorithms for creating smooth curves and surfaces with efficient mathematical representations (Khan, 2018). Trigonometric Bézier curves, a generalization of classical Bézier curves, have been used in various domains such as computer graphics, image processing, and geometric modelling due to their flexibility and precision (Maqsood et al., 2020). This study employs two types of trigonometric Bézier curves: Quasi-Quartic and Quasi-Quintic. Bézier curves are advantageous due to their ease of computation, stability at lower control point degrees, and ability to be manipulated through point operations. Despite the

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existence of newer algorithms with complex formulas, Bézier curves remain widely used in CAGD applications as the newer methods are not always suitable for curve construction (Khan, 2018). In the context of Arabic calligraphy, these curves effectively capture the intricate details and fluidity of the script (Moustapha & Krishnamurti, 2001).

Arabic calligraphy has persisted as a classic art style that is closely associated with Arab and Islamic history. Arabic script, which has historically been produced by expert artisans who painstakingly fashioned complex letterforms, is an essential part of architectural legacy. It is an expressive design style that blends geometric principles with cultural language, allowing letters to be stretched and reshaped in a variety of ways to generate a wide range of patterns (Hussein, 2017). The challenge in digital redesign lies in replicating the precision and artistry inherent in the curvature of Arabic calligraphy. The use of Computer-Aided Geometric Design (CAGD) in redesigning Arabic calligraphy offers several advantages, including enhanced precision, scalability, accessibility, and the ability to merge traditional elements with modern techniques. Arabic calligraphy, a significant aspect of Islamic art and culture, has garnered interest from many researchers due to its aesthetic and geometric complexities (Moustapha & Krishnamurti, 2001). The reconstruction of Arabic calligraphy using mathematical models has been extensively explored, with Trigonometric Bézier curves emerging as a promising approach (Maqsood et al., 2020).

This paper aims to delve deeper into the utilization of Trigonometric Bézier curves for reconstructing Arabic calligraphy. It applies two methods of Trigonometric Bézier Curves—Quasi-Quartic (Yang et al., 2012) and Quasi-Quintic (Misro et al., 2017)—to this purpose. The study examines the elements that influence curve modification outcomes by varying shape parameters. Both methods are assessed by comparing the smoothness, precision, and consistency of the resulting figures against the actual image, as well as evaluating the CPU time required to create the entire Arabic calligraphy.

2. METHODOLOGY

This paper presents the design of a two-dimensional shape using the Quasi-Quartic and Quasi-Quintic Trigonometric Bézier curves method. The figures were created and visualized using Mathematica software.

2.1 Quasi-Quartic Trigonometric Bézier curves

Quartic itself is an algebraic equation or function of the fourth degree. Under the appropriate circumstances, Quasi-Quartic trigonometric Bézier curves not only inherit most of the properties of Quartic Bézier curves, but it can also express any plane or space curve defined by a parametric equation, including some quadratic curves such as circular arcs, parabolas, cardioid exactly, and circular helix (Yang et al., 2012). The introduced base functions define the Quasi-Quartic Trigonometric polynomial base function with a shape parameter λ over the space Ω=span $\{1, \sin t, \cos t, \sin 2t, \cos 2t\}$, as well as with the corresponding Quasi-Quartic Trigonometric Bézier curves and surfaces. This method can adjust its shapes locally or totally when compared with Quartic Bézier curves (Yang et al., 2012).

Quasi-Quartic Trigonometric polynomial Bézier curves can precisely represent straight line segments, circular segments, elliptic segments, parabolas, and cardioids. Quadratic surfaces that can be represented by Bézier surfaces are cylindrical surfaces, spheres, ellipsoids, parabolic surfaces, and toruses. According to Sharma (2016), shape parameter λ and μ influence the control point curve. The shape parameters λ and μ help to affect local control in the curve; λ increases, the curve moves in the direction of control points and as λ decreases, the curve moves in the opposite direction to the control points. Furthermore, Yang (2012) states that some complex surfaces can be precisely constructed by these basic surfaces. The larger is parameter λ, and more approach is the Quasi-Quartic Trigonometric Bézier curve to the control polygon.

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Based on Yang et al. (2012), for $t \in \left[0, \frac{\lambda}{2}\right]$, $b_{0,4}(t), b_{1,4}(t), b_{2,4}(t), b_{3,4}(t)$, and $b_{4,4}(t)$ are called Quasi-Quartic Trigonometric polynomial base functions with a shape parameter λ which can be defined below:

$$\begin{cases} b_{0,4}(t) = \left(1 + \frac{\lambda}{2}\right) - (1+\lambda)\sin t - \frac{\lambda}{2}\cos 2t \\ b_{1,4}(t) = (1+\lambda)(-\frac{3}{2} + 2\sin t + \cos t - \frac{1}{2}\sin 2t + \frac{1}{2}\cos 2t) \\ b_{2,4}(t) = 2(1+\lambda)(1 - \sin t - \cos t + \frac{1}{2}\sin 2t \\ b_{3,4}(t) = (1+\lambda)(-\frac{3}{2} + \sin t + 2\cos t - \frac{1}{2}\sin 2t - \frac{1}{2}\cos 2t \\ b_{4,4}(t) = \left(1 + \frac{\lambda}{2}\right) - (1+\lambda)\cos t + \frac{\lambda}{2}\cos 2t \end{cases} \quad \text{where } -1 \le \lambda \le 1.5 \qquad (1)$$

2.1.1 The Equation of the Quasi-Quartic Trigonometric Bézier curves

For five control points P_i, ($i = 0,1, \dots ,4$), are given, $t \in \left[0, \frac{\pi}{2}\right]$, we define the curve as:

$$\mathrm{B(t)} = \sum_{i=0}^{4} P_i b_{i,4}(t) \qquad (2)$$

The basis graph of the Quasi-Quartic curves with the different value of shape parameters, λ is presenting in Fig. 1.

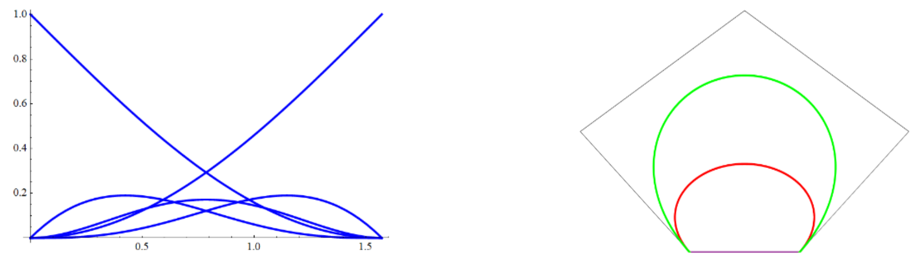

Fig. 1. (left) Quasi-Quartic Trigonometric Bézier basis functions with λ=0; (right) Quasi-Quartic Trigonometric Bézier curves with different λ = -0.9 (purple), 0 (red), 1.5 (green)

2.2 Quasi-Quintic Trigonometric Bézier curves

Quasi-Quintic refers to the fifth degree of a trigonometric Bézier curve equipped with two shape parameters. Compared to a standard Bézier curve, these shape parameters offer enhanced control over the curve's shape. Consequently, the inclusion of shape parameters allows for increased flexibility in the curve's configuration without altering its control points, a technique pivotal in curve and surface construction. Moreover, adjusting the shape parameters, rather than the control points, preserves the curve's geometric characteristics. However, to ensure smoothness, both parametrically and geometrically, when interpolating curves or surface patches between points, specific continuity constraints must be satisfied (Misro et al., 2017). In addition to the traditional Quintic trigonometric Bézier curve, a new Quasi-Quintic trigonometric Bézier curve with two shape parameters has been proposed. This curve retains all the geometric properties

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of the traditional Bézier curve and has been utilized in constructing both open and closed curves (Bashir et al., 2013).

A class of Quasi-Quintic Trigonometric Bézier curves with properties like classical Quintic Bézier curves is proposed. The resulting Quasi-Quintic Trigonometric Bézier curves can be flexibly adjusted for the same control points by changing the values of two shape parameters without changing their control points. A newly constructed quasi-quintic Trigonometric Bézier curve with two shape parameters that inherits most of the geometric properties of the traditional Bézier curve. According to Bashir et al., the values of the shape parameters can be used to change the shape of the curve. The proposed curve can generate both open and closed curves. Under the right conditions, the curve can be used to generate Trigonometric curves such as circle arcs, ellipses, and parabolas. It can also extend to tensor product surfaces. Tan and Zhu (2019) state that the researchers can change the shape by simply changing the values of the two shape parameters. It is because Quasi-Quintic Trigonometric Bézier curves can outline the shape of the control polygon much more precisely. Bashir et al. (2013) states that after the control points have been selected, the Bézier curve or surface is generated uniquely by applying the Bernstein basis functions and that the shape is not adjustable in any way. The introduction of shape parameters has corrected this flaw. To accomplish local control, the values of the shape parameters λ and μ can be employed.

Based on Zhu et al., (2012), for $t \in \left[0, \frac{\pi}{2}\right]$, the following six functions are defined as Quasi-Quintic Trigonometric blending functions

$$\begin{cases} b_0(t) = (1-\sin t)^5(1-\lambda\sin t) \\ b_1(t) = \sin t\,(1-\sin t)^4[5+\lambda(1-\sin t)], \\ b_2(t) = \frac{1}{7}(1-\sin t)^2\,(1-\cos t)[74+(94\sin t+17)(1-\cos t)+66\cos^2 t], \\ b_3(t) = \frac{1}{7}(1-\cos t)^2\,(1-\sin t)[74+(94\cos t+17)(1-\sin t)+66\sin^2 t], \\ b_4(t) = \cos t\,(1-\cos t)^4[5+\mu(1-\cos t)], \\ b_5(t) = (1-\cos t)^5(1-\lambda\cos t), \end{cases} \tag{3}$$

where λ, μ are shape parameters and $-5 \leq \lambda,\ \mu \leq 1$

2.2.1 The Equation of the Quasi-Quintic Trigonometric Bézier curves

For the control points P_i, $(i = 0,1, \dots, n)$, in $\mathbb{R}^2$, $r(t)$ we define the curve as:

$$f(u) = \sum_{i=0}^{n} f_i(u)P_i , u \in \left[0, \frac{\pi}{2}\right],\ \lambda,\ \mu\ \in [-5,1] \tag{4}$$

For the control points $f_i(u)$, $i = 0, 1, \dots, n$ in $\mathbb{R}^2$ are the Trigonometric basis function.

The basis graph of the Quasi-Quintic curves with the different value of shape parameters, λ is presenting in Fig. 2.

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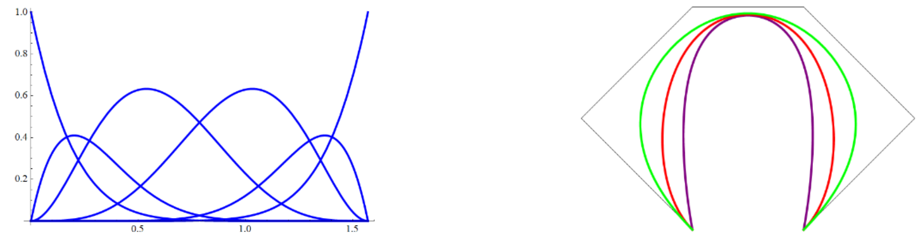

Fig. 2. (left) Quasi- Quintic Trigonometric Bézier basis functions with λ= μ =0; (right) Quasi- Quintic Trigonometric Bézier curves with different λ= μ = -5(purple), -2 (red), 1 (green)

3. APPLICATION IN RECONSTRUCTION OF THE ARABIC CALLIGRAPGY OUTLINE

In this paper, both methods are employed to reproduce the outlines of the Thuluth script in Arabic calligraphy. The word غروب (pronounced as "ghrub"), meaning "sunset," is chosen as the reference figure, as shown in Fig. 3.

Fig 3. Actual reference figure of Arabic calligraphy
(Source: https://www.arabiccalligraphygenerator.com/)

This paper compares two methods for reconstructing the outline of Arabic calligraphy, as shown in Fig. 3. Both methods utilize shape parameters but operate over different interval ranges. The study focuses on global changes in all contours by applying the same parameter values to each curve. Three parameter values—selected from the lowest, moderate, and highest points in their ranges—were used in the study, chosen for their distinct effects on the resultant images. Even minor changes in shape parameters result in slight variations in the contour images.

The reconstructed image was generated and analysed for precision in contour matching compared to the reference figure. Additionally, the performance of each method was evaluated based on the CPU time required to generate the entire image.

Based on the reference figure, the word was divided into 69 curves. Data points were assigned according to the respective curve degrees: 5 data points for each curve in the Quasi-Quartic method and 6 data points for each curve in the Quasi-Quintic method. Using these data points, both methods were employed to recreate the outlines of the figure, as detailed in Table 1 and 2.

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Table 1. Result of shape design for Quasi-Quartic Trigonometric Bézier

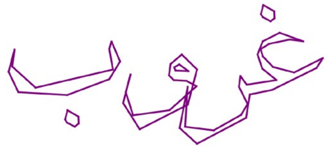
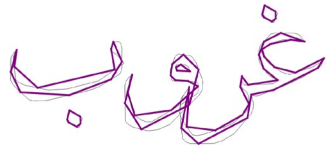
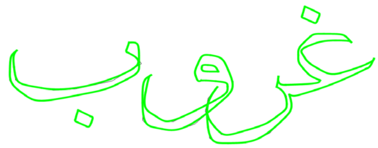
λ	Without control polygon	With control polygon
-1		
0.25		
1.5		

Table 1 present the outcomes from combining 69 curves with varying shape parameters using each method. For the Quasi-Quartic curves, the parameters used are λ = -1, 0.25, and 1.5. A detailed side-by-side comparison of all the results is made against the reference figure, assessing each curve's contours. Based on the visual comparison, λ = -1 and λ = 1.5 create uneven contours that differ from the reference figure, while the parameter of 0.25 accurately reproduces the precise contours of the reference figure, yielding smooth curves.

Table 2. Result of shape design for Quasi-Quintic Trigonometric Bézier

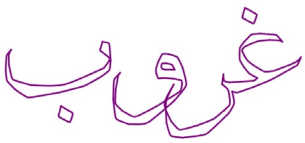
λ, μ	Without control polygon	With control polygon
-5		
-2		
1		

Table 2 display the reconstructed image using the Quasi-Quintic method with parameters λ, μ = -5, -2, and 1. A comprehensive side-by-side comparison of all the results is made against the actual figure, assessing the contours of each curve in detail. The findings indicate that using intermediate values within

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the parameter range effectively reproduces the precise contours of the actual figure, resulting in smooth curves. In contrast, other parameter values lead to inconsistent curve contours, producing unsatisfactory images.

Table 3. CPU Time of different values of shape parameters

Methods	Degree	Number of control points	Number of curves	Shape Parameter	CPU Time (seconds)
Quasi-Quartic Trigonometric Bézier	4	345	69	$\lambda = -1$	5.483
				$\lambda = 0.25$	8.453
				$\lambda = 1.5$	9.734
Quasi-Quintic Trigonometric Bézier	5	414	69	λ and $\mu = -5$	9.594
				λ and $\mu = -2$	9.344
				λ and $\mu = 1$	9.015

The performance of each method was evaluated by analyzing the CPU time required to generate the entire figures. Table 3 presents the results, showing that the Quasi-Quartic method had the shortest CPU time. Specifically, $\lambda = -1$ required the least time at 5.483 seconds, but the resulting images did not match the reference figure satisfactorily. Conversely, $\lambda = 1.5$ took longer, at 9.734 seconds, and produced unsatisfactory images. However, for precise image reproduction, $\lambda = 0.25$ required 8.453 seconds of CPU time.

For the Quasi-Quintic method, with 414 control points, the CPU time for λ, $\mu = 1$ was significantly less, at about 0.579 seconds, compared to λ, $\mu = -5$. The optimal parameters for producing the best images required 9.344 seconds. Comparing both methods, the average CPU time for the Quasi-Quintic method was 7.89 seconds, whereas for the Quasi-Quartic method, it averaged 9.317 seconds. This indicates that the Quasi-Quintic method demonstrates superior computational performance in reconstructing the entire images.

4. RESULTS AND DISCUSSION

This paper illustrates the reconstruction of Arabic calligraphy outlines using Quasi-Quartic and Quintic Trigonometric Bézier Curves. After analyzing the resulting figures against the actual image, Table 4 presents the images that are comparable for each method to the reference figure, as detailed in Table 1 and 2.

Table 4. Comparison of outline designs with the actual image

Reference Figure	Quasi-Quartic Trigonometric Bézier	Quasi-Quintic Trigonometric Bézier
		
Shape Parameter	$\lambda = 0.25$	λ and $\mu = -2$
CPU Time (seconds)	8.453	9.344

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Displayed in Table 4, the comparison between the methods was conducted by assessing the accuracy in reproducing the reference image through smooth contouring and the CPU performance in reconstructing the entire figure. Upon examination of both sets of figures, it becomes apparent that the Quasi-Quintic method produced uneven curves for the entire image, unlike the Quasi-Quartic method, which generated more precise, consistent, and smoother contours. These findings lead to the conclusion that the Quasi-Quartic curve emerges as the optimal method for recreating the smoothest outlines, requiring the least CPU time of 8.453 seconds.

5. CONCLUSION

Analysing and comparing CAGD methods is vital for comprehending the diverse strengths and limitations they entail. Incorporating Quasi Trigonometric Bezier curves into the revamping of Arabic calligraphy presents promising avenues for upholding tradition, refining precision, and fostering creative exploration. Further research and exploration in this domain are indispensable for advancing the utilization of CAGD methods in the redesign of Arabic calligraphy while maintaining due respect for its cultural significance.

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7. CONFLICT OF INTEREST STATEMENT

The authors agree that this research was conducted in the absence of any self-benefits, commercial or financial conflicts and declare the absence of conflicting interests with the funders.

8. AUTHORS' CONTRIBUTIONS

Noor Khairiah Razali: Conceptualisation, methodology, formal analysis supervision, writing- review and editing, and validation; **Athirah Hanani Mohd Fauzi**: Conceptualisation, methodology, formal analysis, investigation and writing-original draft, formal analysis, and validation.

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