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A Comparative Study of Backward Euler and Adams-Moulton Methods for Lotka-Volterra Prey-Predator Model

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ABSTRACT

This research investigates the comparison between the backward Euler and Adams-Moulton methods in solving the Lotka-Volterra prey predator model, specifically analyzing the interaction between wolf and moose populations. The study aims to identify which numerical method provides a more accurate approximation of the model's solutions. Data collected from the interactions between wolves and moose on Isle Royale from 1959 to 2019 was used, determining a carrying capacity of 21.24 for wolves and 948.15 for moose. When the initial population is below this carrying capacity, it tends to grow due to the availability of adequate resources. The comparison results revealed that the Adams-Moulton method provided the most accurate approximation, successfully achieving the primary objective of the research. The equilibrium and stability of the system were assessed by defining its dynamics through mathematical equations and evaluating the eigenvalues of the Jacobian matrix, resulting in a growth rate value of 0.5017. The system was found to be stable when the populations of moose and wolves oscillated with consistent amplitude, influenced by the growth rate. The findings emphasize the importance of carrying capacity and initial conditions in understanding equilibrium and stability in prey-predator interactions, contributing to population dynamics. This research aids in the development of effective conservation and management strategies for maintaining ecosystem balance.

1. INTRODUCTION

The Lotka-Volterra model, established in the 1920s, describes the dynamics of prey predator interactions through differential equations (Anisiu & Academy, 2014). This research aims to compare the backward Euler and Adams-Moulton numerical methods in modelling these interactions, specifically focusing on the cyclical behavior, equilibrium, and stability of the prey predator relationship. The primary objectives are to investigate the interactions using these methods and to determine which method provides more accurate and efficient solutions. The novelty of this research lies in the comparative analysis of these

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two implicit numerical methods, providing insights into their effectiveness in ecological modelling, which can aid in developing better conservation strategies.

Zayernouri and Matzavinos (2016) highlighted that the Adams-Moulton method, an implicit technique used to solve ordinary differential equations, is particularly effective for modeling dynamic systems like the Keller-Segel chemotaxis system. They noted that this method produces more precise outcomes and is more efficient in terms of computational resources—such as time and memory—compared to conventional methods when applied to systems described by fractional differential equations.

The Backward Euler method is a numerical technique used to solve ordinary differential equations (ODEs) by using an initial value to predict the system's future state. This method is particularly useful for analyzing how systems change over time by focusing on the current rate of change. It also one of several methods available for solving differential equations.

The combination of these methods allows for a comprehensive analysis of the dynamics of predator-prey interactions, enabling the evaluation of equilibrium points and stability, which are crucial for understanding population dynamics and developing effective conservation strategies.

2. LITERATURE

Files should be in MS Word format only and should be formatted for direct printing. Figures and tables should be embedded and not supplied separately. Numerical approximation methods are commonly used to solve the Lotka-Volterra equations for studying prey-predator dynamics (Laham et al., 2012; Elsadany & Matouk, 2014). Paul et al. (2016) compared the Runge-Kutta-Fehlberg (RKF) method and the Laplace Adomian Decomposition method (LADM) by plotting their numerical solutions, which highlighted differences and accuracies in approximating ecological behaviors. Similarly, Manaf et al. (2023) and Rahaman et al. (2024) investigated the effectiveness of the Euler method, Taylor Series method and RKF method for the Lotka-Volterra competitive model, using graphical representations to show that RKF provided more accurate results compared to the Taylor Series method. Both studies utilized visual comparisons to assess the performance of different numerical methods in approximating solutions to the Lotka-Volterra model, enhancing the understanding of population dynamics in ecological systems.

The Lotka-Volterra competition model describes the dynamics of two species competing for limited resources, characterized by oscillations in their populations. This model provides insights into interspecific and intraspecific competition within ecological communities, influenced by factors like competition coefficients and carrying capacity (Razali & Abdullah, 2013). Numerical methods such as the Taylor Series and Runge-Kutta methods are employed to approximate solutions to the differential equations governing these population dynamics. The Taylor Series method uses derivatives to estimate function values iteratively, while the Runge-Kutta method adjusts step sizes based on truncation errors to provide accurate approximations without requiring higher-order derivatives.

The logistic equation models carrying capacity explained by Al-Moqbali et al. (2018) stated that the sigmoidal growth of populations influenced by environmental changes. Their study examines models with variable carrying capacity and Holling type I and II functional responses, revealing that variable carrying capacity significantly affects prey-predator dynamics, leading to damped oscillations and stable equilibria where both populations coexist. The article highlights the importance of incorporating variable carrying capacity in ecological models to understand population dynamics better.

Vaidyanathan (2015) explores the Lotka-Volterra model with negative feedback, noting that in the absence of predators, prey populations grow to a stable carrying capacity. Conversely, without prey, predators decline and face extinction. However, both populations can coexist stably if certain conditions are met, specifically if the prey's growth rate exceeds the predator's death rate and the predator's growth rate surpasses the prey's death rate. Other than that, prey refuges are also important for system stability. The research indicates that under certain conditions, the system can exhibit globally asymptotic stability at

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specific equilibrium points, particularly when the intrinsic growth rate of prey is low and the prey refuge is sufficiently high (Majeed & Ghafel, 2022). This suggests that prey fear can enhance stability at positive equilibrium points, leading to a more resilient ecosystem.

3. METHODOLOGY

The research used data from Vucetich and Peterson (2011) to study the interactions between moose and wolves on Isle Royale, Lake Superior, from 1959 to 2019. It focuses on how wolves, as predators, might affect moose populations and their distribution. The initial populations were 788 moose and 50 wolves. Population growth was tracked through annual counts of living moose and wolves in the study area during this period.

The Lotka-Volterra model examines prey-predator dynamics, proposing that both species can achieve a dynamic equilibrium with their populations fluctuating in response to each other. The model includes two key equations: one for prey growth and predation, and one for predator growth and mortality. Parameters in these equations represent the prey's growth rate, the predation rate, the predator's death rate, and the growth rate of predators due to consuming prey.

The equations are generally expressed as follows:

Prey Equation:

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad (1)$$

Predator Equation:

$$\frac{dy}{dt} = \delta xy - \gamma y \qquad (2)$$

where:

x represents the prey population,

y represents the predator population,

t is time,

$\alpha, \beta, \gamma, and\ \delta$ are rates that determine the interactions between the prey and predator.

The equation's terms can be interpreted as follows:

α represents the prey's growth rate,

β represents the rate that predators capture and consume prey,

γ represents the predator's death rate,

δ represents the rate that predators grow by consuming prey.

The backward Euler and Adams-Moulton methods will be used to simulate prey-predator interactions, and these results will be compared to simulation data. Using these numerical methods helps systematically analyze and understand the dynamics of prey and predator populations over time.

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3.1 Exact Solution

The logistic model describes population growth through a logistic equation, while the exponential growth function is expressed as follows:

$$\frac{dy}{dt} = ry. \quad (3)$$

The answer for the initial condition can be derived by solving equation (3), as follows:

$$y(t) = \frac{k}{1 + \frac{1}{y_0} e^{-rt}(k - y_0)} \quad (4)$$

The best fit can be achieved by using curve fitting with the parameters r and k, which ensures the logistic equation closely matches the observed data for both wolves and moose. The results are shown in Fig. 1 and Fig. 2.

The best fit for wolve is $r = 0.9$ and $k = 21.24$; The best fit for moose is $r = 0.7$ and $k = 948.15$;

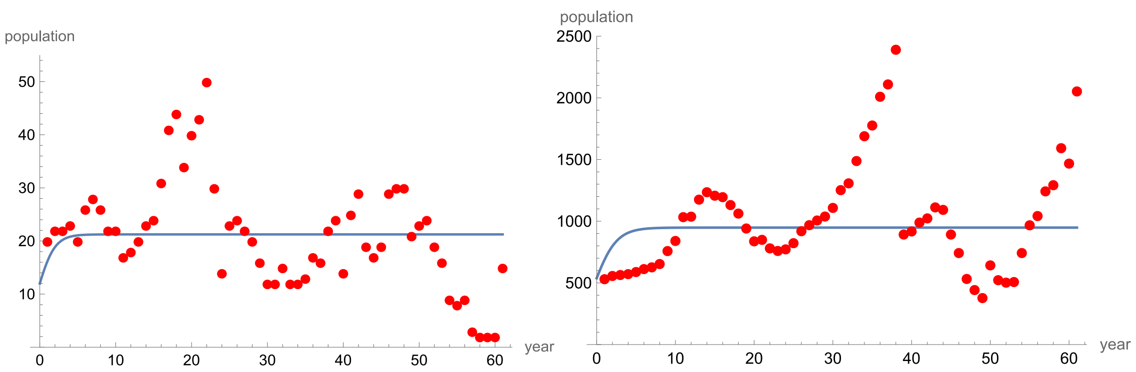

Fig. 1. Curve fitting for growth of wolve (left) and moose (right)

Based on observations of Fig. 1 and Fig. 2 above, wolves produce a carrying capacity of 21.24, while moose produce a carrying capacity of 948.15. From the best fit curve fitting, the logistic equation was utilized to model the growth of the wolf and moose populations.

For wolve:

$$\frac{dy}{dt} = 0.9y\left(1 - \frac{y}{21.24}\right) \quad (5)$$

with $y_0 = 20$, where y is the mean density at time t (in years), then, the logistic equation as;

$$y(t) = \frac{21.24}{1 + 0.062e^{-0.9t}} \quad (6)$$

For moose:

$$\frac{dy}{dt} = 0.7y\left(1 - \frac{y}{948.15}\right) \quad (7)$$

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with $y_0 = 538$, where y is the mean density at time t (in years), then, the logistic equation as;

$$y(t) = \frac{948.15}{1 + 0.76236e^{-0.7t}} \tag{8}$$

3.2 Backward Euler Method

The Backward Euler formula is expressed as $y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$. For the initial values, $x_0 = 0$ and the initial population of wolf is $y_0 = 20$ with a step size $h = 1$, the general equation for the Backward Euler method is:

$$y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$$

$$y_{n+1} = y_n + (1)\left(0.9y_{n+1}\left(1 - \frac{y_{n+1}}{21.24}\right)\right)$$

$$y_{n+1} = y_n + 0.9y_{n+1} - 0.042373(y_{n+1})^2$$

$$\therefore 0.042373(y_{n+1})^2 + 0.1y_{n+1} - y_n = 0$$

The same calculation will be continued for another species which is moose with initial value is $y_0 = 538$.

3.3 Adams-Moulton Method

The second-order Adams-Moulton method is well-suited for solving stiff differential equations.

The calculation as below;

To find y_1 :

$Y_1 = y_0$

$K_1 = f(x_0, y_0)$

$Y_2 = y_0 + hF_1$

$K_2 = f(x_1, Y_2)$

$y_1 = y_0 + \frac{h}{2}K_1 + \frac{h}{2}K_2$

To find y_2 :

$K_1 = f(x_0, y_0)$

$K_2 = f(x_1, y_1)$

$ab_{new} = y_1 + \frac{h}{2}(3K_2 - K_1)$

$K_3 = f(x_2, ab_{new})$

$am_{new} = y_2 = y_1 + \frac{h}{2}K_3 + \frac{h}{2}K_2$

Since the problem in this case takes a long time to solve using a precise analytical solution, Wolfram Mathematica 13.2 software is used to solve it numerically.

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4. RESULTS AND DISCUSSIONS

This research aims to compare the Lotka-Volterra prey-predator interactions using the backward Euler and Adams-Moulton methods, focusing on the cyclical dynamics of two species over time and analyzing the equilibrium and stability of their relationship based on initial population values. This analysis is intended to enhance understanding of population dynamics and the effects of predation within ecological systems.

Parameter estimation for the Logistic model is done using curve fitting techniques to find the growth rate r and carrying capacity k that best match observed population data. This is achieved by minimizing the difference between observed data and model predictions. For the wolf population, the best fit values were $r = 0.9$ and $k = 21.24$, using the logistic equation:

$$y(t) = \frac{k}{1 + \frac{1}{y_0} e^{-rt}(k - y_0)}$$

where y_0 is the initial population size. The report doesn't mention the r^2 value, which measures how well the model fits the data, but it may be found in the curve fitting analysis sections.

The logistic model is used in this study to understand how wolf and moose populations grow over time, considering the environment's carrying capacity. It helps predict long-term population behavior and interactions between these species. For wolves, the growth rate is $r = 0.9$ and the carrying capacity is $k = 21.24$, while for moose, $r = 0.7$ and $k = 948.15$. This model is important for predicting future population sizes, assessing stability, and informing wildlife management and conservation efforts.

Table 1. Exact solution and the numerical solution for wolve

Year	Exact Solution	Backward Euler	Adams-Moulton
0	20.00000	20.00000	20.00000
1	20.71776	20.57758	20.60978
2	21.02453	20.88861	21.03120
3	21.15187	21.05429	21.21554
4	21.20408	21.14204	21.26441
5	21.22538	21.18838	21.26222
6	21.23405	21.21282	21.25077
7	21.23758	21.22569	21.24312
8	21.23902	21.23247	21.24002
9	21.23960	21.23603	21.23938
10	21.23984	21.23791	21.23956
11	21.23993	21.23890	21.23981
12	21.23997	21.23942	21.23996
13	21.23999	21.23970	21.24001
14	21.24000	21.23984	21.24001

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15	21.24000	21.23992	21.24001
16	21.24000	21.23996	21.24000
17	21.24000	21.23998	21.24000
18	21.24000	21.23999	21.24000
19	21.24000	21.23999	21.24000
20	21.24000	21.24000	21.24000
21	21.24000	21.24000	21.24000
22	21.24000	21.24000	21.24000
23	21.24000	21.24000	21.24000
24	21.24000	21.24000	21.24000
25	21.24000	21.24000	21.24000
26	21.24000	21.24000	21.24000
27	21.24000	21.24000	21.24000
28	21.24000	21.24000	21.24000
29	21.24000	21.24000	21.24000
30	21.24000	21.24000	21.24000
31	21.24000	21.24000	21.24000
32	21.24000	21.24000	21.24000
33	21.24000	21.24000	21.24000
34	21.24000	21.24000	21.24000
35	21.24000	21.24000	21.24000
36	21.24000	21.24000	21.24000
37	21.24000	21.24000	21.24000
38	21.24000	21.24000	21.24000
39	21.24000	21.24000	21.24000
40	21.24000	21.24000	21.24000
41	21.24000	21.24000	21.24000
42	21.24000	21.24000	21.24000
43	21.24000	21.24000	21.24000
44	21.24000	21.24000	21.24000
45	21.24000	21.24000	21.24000
46	21.24000	21.24000	21.24000
47	21.24000	21.24000	21.24000
48	21.24000	21.24000	21.24000
49	21.24000	21.24000	21.24000
50	21.24000	21.24000	21.24000

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51	21.24000	21.24000	21.24000
52	21.24000	21.24000	21.24000
53	21.24000	21.24000	21.24000
54	21.24000	21.24000	21.24000
55	21.24000	21.24000	21.24000
56	21.24000	21.24000	21.24000
57	21.24000	21.24000	21.24000
58	21.24000	21.24000	21.24000
59	21.24000	21.24000	21.24000
60	21.24000	21.24000	21.24000

Table 2. Exact solution and the numerical solution for moose

Year	Exact Solution	Backward Euler	Adams-Moulton
0	538.00018	538.00000	538.00000
1	687.77454	674.32205	683.42414
2	798.10898	773.88657	793.39823
3	867.19248	840.62069	865.96710
4	906.14202	883.05739	908.40535
5	926.81357	909.20143	930.59422
6	937.43324	925.00646	941.04144
7	942.79776	934.45470	945.50461
8	945.48458	940.06556	947.24657
9	946.82452	943.38458	947.86985
10	947.49132	945.34339	948.07353
11	947.82280	946.49787	948.13326
12	947.98749	947.17775	948.14819
13	948.06929	947.57794	948.15084
14	948.10992	947.81345	948.15079
15	948.13010	947.95201	948.15042
16	948.14012	948.03353	948.15018
17	948.14509	948.08149	948.15007
18	948.14756	948.10970	948.15003
19	948.14879	948.12629	948.15001
20	948.14940	948.13605	948.15000
21	948.14970	948.14180	948.15000
22	948.14985	948.14517	948.15000

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23	948.14993	948.14716	948.15000
24	948.14996	948.14833	948.15000
25	948.14998	948.14902	948.15000
26	948.14999	948.14942	948.15000
27	948.15000	948.14966	948.15000
28	948.15000	948.14980	948.15000
29	948.15000	948.14988	948.15000
30	948.15000	948.14993	948.15000
31	948.15000	948.14996	948.15000
32	948.15000	948.14998	948.15000
33	948.15000	948.14999	948.15000
34	948.15000	948.14999	948.15000
35	948.15000	948.15000	948.15000
36	948.15000	948.15000	948.15000
37	948.15000	948.15000	948.15000
38	948.15000	948.15000	948.15000
39	948.15000	948.15000	948.15000
40	948.15000	948.15000	948.15000
41	948.15000	948.15000	948.15000
42	948.15000	948.15000	948.15000
43	948.15000	948.15000	948.15000
44	948.15000	948.15000	948.15000
45	948.15000	948.15000	948.15000
46	948.15000	948.15000	948.15000
47	948.15000	948.15000	948.15000
48	948.15000	948.15000	948.15000
49	948.15000	948.15000	948.15000
50	948.15000	948.15000	948.15000
51	948.15000	948.15000	948.15000
52	948.15000	948.15000	948.15000
53	948.15000	948.15000	948.15000
54	948.15000	948.15000	948.15000
55	948.15000	948.15000	948.15000
56	948.15000	948.15000	948.15000
57	948.15000	948.15000	948.15000
58	948.15000	948.15000	948.15000

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59	948.15000	948.15000	948.15000
60	948.15000	948.15000	948.15000

Table 1 and Table 2 shows the comparison between the backward Euler and Adams-Moulton methods and the exact solution for wolves. Graphs were plotted from the table to make the comparison clearer.

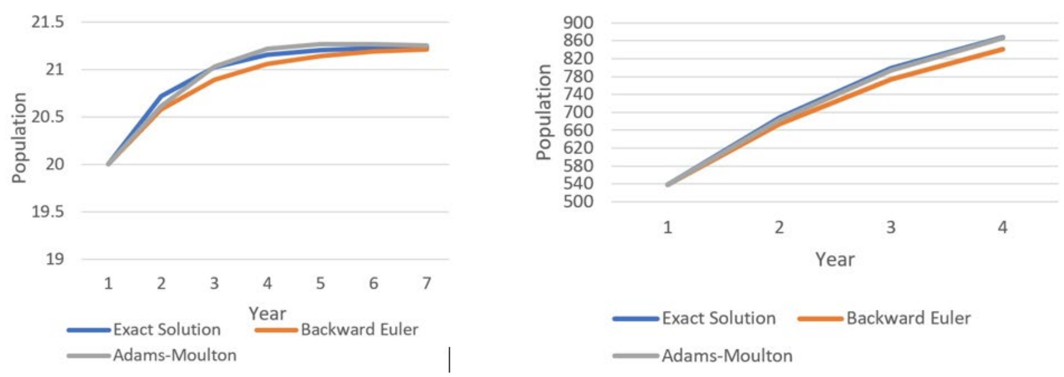

Fig. 2. Comparison between numerical methods and exact solution of wolve (left) and moose (right)

Fig. 3 and Fig. 4 presents a comparative analysis of the numerical methods used to approximate the moose and wolf population dynamics in the Lotka-Volterra model, respectively, showcasing the results from both the backward Euler and Adams-Moulton methods alongside the exact solution. This graph likely includes numerical values that highlight the accuracy of each method in predicting population changes over time. The Adams-Moulton method is deemed the best method in this research due to its superior accuracy, as it consistently yields results that are closer to the exact solution compared to the backward Euler method. Additionally, the Adams-Moulton method is recognized for its stability, particularly in handling nonlinear problems like those presented in the Lotka-Volterra equations, which is essential for maintaining realistic population values in oscillatory dynamics. Furthermore, it may also offer greater computational efficiency, requiring fewer steps to achieve a similar level of accuracy. Collectively, these advantages underscore the effectiveness of the Adams-Moulton method in modelling the interactions between wolves and moose, making it the preferred choice for this ecological study.

Understanding prey-predator interactions is important for ecological modeling and management. This research focuses on the dynamics between moose and wolves, aiming to compare the accuracy of two numerical methods: backward Euler and Adams-Moulton. To evaluate these methods, we not only compare their numerical results but also analyze the system's equilibrium points and stability. Stability analysis helps predict whether populations will stay constant or return to equilibrium after a disturbance. Using the Jacobian method, we assessed the stability of equilibrium points by checking the eigenvalues, which indicated whether the populations would return to equilibrium or move away from it.

At the critical point $(0,0)$, the Jacobian matrix has eigenvalues $r = \alpha$ and $r = -\gamma$, meaning this point is a saddle point, with one positive and one negative eigenvalue. At the critical point $\left(\frac{\gamma}{\delta}, \frac{\alpha}{\beta}\right)$, the eigenvalues are purely imaginary, given by $r = \pm\sqrt{\alpha\gamma}\, i$, which indicates oscillatory behavior around this equilibrium point. Table 3 below displays the results of the Jacobian method using Wolfram Mathematica 13.2 software.

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Table 3. Numerical solution for wolve and moose

Year	Numerical solution		Year	Numerical solution	
	Moose in mixed	Wolves in mixed		Moose in mixed	Wolves in mixed
0	538.00000	20.00000	31	1049.86047	19.50169
1	554.51163	19.98393	32	1066.37209	19.48561
2	571.02326	19.96785	33	1082.88372	19.46954
3	587.53488	19.95178	34	1099.39535	19.45346
4	604.04651	19.93570	35	1115.90698	19.43739
5	620.55814	19.91963	36	1132.41860	19.42131
6	637.06977	19.90355	37	1148.93023	19.40524
7	653.58140	19.88748	38	1165.44186	19.38916
8	670.09302	19.87140	39	1181.95349	19.37309
9	686.60465	19.85533	40	1198.46512	19.35702
10	703.11628	19.83925	41	1214.97674	19.34094
11	719.62791	19.82318	42	1231.48837	19.32487
12	736.13953	19.80710	43	1248.00000	19.30879
13	752.65116	19.79103	44	1264.51163	19.29272
14	769.16279	19.77496	45	1281.02326	19.27664
15	785.67442	19.75888	46	1297.53488	19.26057
16	802.18605	19.74281	47	1314.04651	19.24449
17	818.69767	19.72673	48	1330.55814	19.22842
18	835.20930	19.71066	49	1347.06977	19.21234
19	851.72093	19.69458	50	1363.58140	19.19627
20	868.23256	19.67851	51	1380.09302	19.18020
21	884.74419	19.66243	52	1396.60465	19.16412
22	901.25581	19.64636	53	1413.11628	19.14805
23	917.76744	19.63028	54	1429.62791	19.13197
24	934.27907	19.61421	55	1446.13953	19.11590
25	950.79070	19.59813	56	1462.65116	19.09982
26	967.30233	19.58206	57	1479.16279	19.08375
27	983.81395	19.56599	58	1495.67442	19.06767
28	1000.32558	19.54991	59	1512.18605	19.05160
29	1016.83721	19.53384	60	1528.69767	19.03552
30	1033.34884	19.51776	61	1545.20930	19.01945

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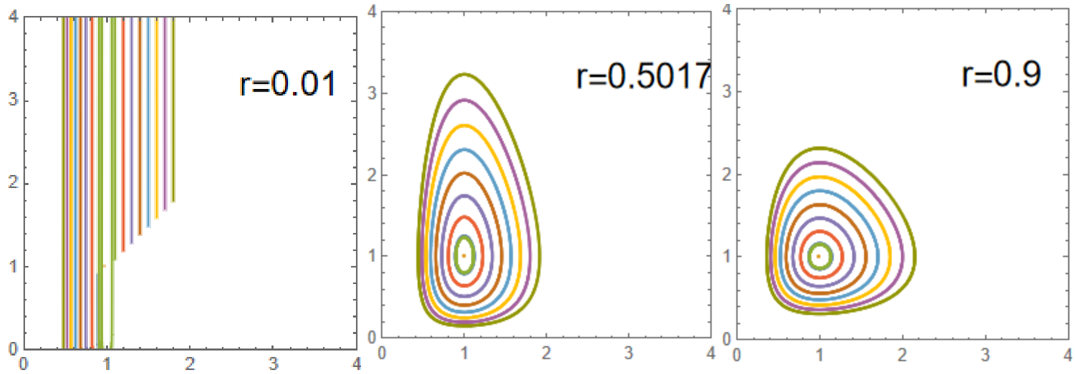

Fig. 3. Phase diagram

The three phase diagrams illustrate the predator-prey interactions under different values of the parameter r, with the x-axis representing the prey population and the y-axis representing the predator population. The actual value $r = 0.5017$ was calculated, while $r = 0.01$ and $r = 0.9$ were assumed for comparison to understand how the system's behavior and stability change under different conditions. Based on the Fig. 3, where $r = 0.01$, the system shows unstable or non-cyclic behavior. The prey population remains relatively steady while the predator population exhibits significant fluctuations. This suggests that the system is unstable and sensitive to changes in initial conditions, leading to divergent behavior without stable oscillations.

In contrast, the actual value of $r = 0.5017$, shows neutral stability. The closed loops in this phase diagram represent limit cycles, indicating that the populations of prey and predators undergo regular, sustained oscillations. This behavior is characteristic of a stable predator-prey interaction where neither population grows uncontrollably or faces extinction.

The last diagram, with $r = 0.9$, also shows limit cycles, but the oscillations are tighter and more controlled compared to $r = 0.5017$. This suggests a more stable interaction between the prey and predator populations, with the system exhibiting regular, balanced cycles. The populations in this scenario remain closer to equilibrium, indicating a stronger level of stability within the system.

In summary, as r increases, the system moves from instability at $r = 0.01$ to neutral stability with sustained oscillations at $r = 0.5017$ and $r = 0.9$, with tighter and more controlled cycles as r approaches 0.9. The stability analysis using the Jacobian matrix revealed that the system remained stable, with consistent oscillations in population sizes when applying the Adams-Moulton method, with actual value of $r = 0.5017$. This indicates its superiority in maintaining equilibrium.

5. CONCLUSION AND RECOMMENDATIONS

This research compares the numerical solutions of Lotka-Volterra prey-predator interactions using the backward Euler and Adams-Moulton methods, focusing on wolves and moose on Isle Royale from 1959 to 2019. The study reveals that the carrying capacity for wolves is about 21.24 and for moose is 948.15. When initial populations are below these carrying capacities, they grow until they reach these limits, given ample resources. The Adams-Moulton method provided the best approximation compared to the backward Euler method. The research also analyzed the equilibrium and stability of the system using the Jacobian matrix and eigenvalues, showing that the system's stability depends on the parameters' influence.

The findings highlight the importance of carrying capacity and initial conditions for understanding prey-predator dynamics, contributing valuable insights for developing conservation and management strategies to maintain ecosystem balance. For future research, it's advisable to extend data collection beyond

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2019 to better understand recent changes in wolf and moose populations on Isle Royale, offering insights into long-term ecological impacts and prey-predator dynamics. Including interactions with other species could provide a fuller picture of ecological balance and species interconnections.

Additionally, studying environmental factors like climate change and habitat alterations would improve understanding of their effects on population dynamics, leading to more accurate models. Utilizing advanced numerical methods and machine learning could enhance model precision, while incorporating spatial and temporal dynamics would offer a clearer view of species distribution and movement. Applying these findings to conservation and wildlife management could guide effective policy-making and maintain ecosystem balance.

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7. CONFLICT OF INTEREST STATEMENT

The authors agree that this research was conducted in the absence of any self-benefits, commercial or financial conflicts.

8. AUTHORS' CONTRIBUTIONS

Nurizatul Syarfinas Ahmad Bakhtiar, **Nur Nadia Sajidah** and **Nur Fatihah Fauzi** devised the research concept and designed the technique. **Nur Izzati Khairudin** and **Huda Zuhrah Ab. Halim** collected and assessed the data, respectively. The results were analyzed, and the manuscript written by **Nurizatul Syarfinas Ahmad Bakhtiar** and **Nur Nadia Sajidah**. The final version was reviewed and authorised for submission by all authors.

9. REFERENCES

Al-Moqbali, M., Al-Salti, N., & Elmojtaba, I. (2018). Prey–predator models with variable carrying capacity. *Mathematics*, *6*(6), 102. https://doi.org/10.3390/math6060102

Anisiu, M.-C., & Academy, R. (2014). Lotka, Volterra and their model, 32 (01). https://tinyurl.com/y63wbb75

Elsadany, A. A., & Matouk, A. E. (2014). Dynamical behaviors of fractional-order Lotka–Volterra predator–prey model and its discretization. *Journal of Applied Mathematics and Computing*, *49*(1-2), 269–283. https://tinyurl.com/5acmwthj

Laham, M. F., Krishnarajah, I., & Jumaat, A. K. (2012). A numerical study on predator prey model. In *International Journal of Modern Physics: Conference Series* (Vol. 9, pp. 347-353). World Scientific Publishing Company. https://doi.org/10.1142/S2010194512005417

Majeed, S. J., & Ghafel, S. F. (2022). Stability analysis of a prey-predator model with prey refuge and fear of adult predator. *Iraqi Journal of Science*, 4374–4387. https://www.iasj.net/iasj/download/2868ff3c9c60b840

https://doi.org/10.24191/jcrinn.v10i1.503

Manaf, M. N. A., Fauzi, N. F., Bakhtiar, N. S. A., Khairudin, N. I., & Halim, H. Z. A. (2023). Comparative analysis of Taylor Series and Runge-Kutta Fehlberg methods in solving the Lotka-Volterra competitive model. *Applied Mathematics and Computational Intelligence (AMCI)*, *12*(3), 91-103. https://doi.org/10.58915/amci.v12i3.323

Paul, S., Mondal, S. P., & Bhattacharya, P. (2016). Numerical solution of Lotka Volterra prey predator model by using Runge–Kutta–Fehlberg method and Laplace Adomian decomposition method. *Alexandria Engineering Journal*, *55*(1), 613-617. https://doi.org/10.1016/j.aej.2015.12.026

Rahaman, N. H. A., Bakhtiar, N. S. A., Hajimia, H., Fauzi, N. F., & Khairudin, N. I. (2024) Comparative analysis of Euler and Runge-Kutta Fehlberg methods in solving the Lotka-Volterra competitive model. *Mathematical Scieneces and Informatic Journals (MIJ)*, *5*(2), 95-104. https://ir.uitm.edu.my/id/eprint/106661/1/106661.pdf

Razali, N. S. A. B., & Abdullah, F. A. (2013, April). Numerical methods for competitive hunters model. In *AIP Conference Proceedings*. *American Institute of Physics, 1522*(1), 140-147. https://doi.org/10.1063/1.4801116

Vaidyanathan, S. (2015). Lotka-Volterra population biology models with negative feedback and their ecological monitoring. *International Journal of PharmTech Research CODEN (USA): IJPRIF*, *8*(5), 974–981. https://sphinxsai.com/2015/ph_vol8_no5/2/(974-981)V8N5PT.pdf

Vucetich, J. A., & Peterson, R. O. (2011). The population biology of isle royale wolves and moose: An overview | The wolves and moose of Isle Royale. Isleroyalewolf.org. https://isleroyalewolf.org/data/data/home.html

Zayernouri, M., & Matzavinos, A. (2016). Fractional Adams–Bashforth/Moulton methods: An application to the fractional Keller–Segel chemotaxis system. *Journal of Computational Physics, 317,* 1–14. https://doi.org/10.1016/j.jcp.2016.04.041

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