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Fitting Logistic Growth Model for Soybean Height Using Nonlinear Optimization with Levenberg-Marquardt

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ABSTRACT

This study focuses on modelling soybean growth using the Logistic growth model, a simple sigmoid model with three easily interpreted parameters. The Logistic growth model is also classified as an intrinsically nonlinear model. Parameter estimation was performed using the Nonlinear Least Square (NLS) method with an iterative algorithm, specifically the Levenberg Marquardt algorithm. The results indicated that the maximum height was 21.6543 cm, the intrinsic growth rate was 0.61368, and the parameter controlling the slope of the curve was 10.5104. All parameters in the Logistic growth model were significant to the model based on the t-test results and the model had an adjusted R-squared value of 0.957113. This value indicates that the model can explain 95.71% of the growth patterns observed in the data through the parameters that have been considered.

1. INTRODUCTION

Often unnoticed events occurring in daily life, are among those that can be modeled using a regression equation, a fundamental aspect of regression analysis. Regression analysis consists of two types, linear and nonlinear regression, and one of the nonlinear models is the growth model. Plant growth will form an 'S' shaped curve when plotted on a graph. This curve describes three phases of plants, specifically the logarithmic, linear, and aging phases. In the logarithmic phase, the growth rate is initially slow but increases continuously. The growth rate is directly proportional to the size of the organism. The larger the organism, the faster it grows. In the linear phase, the increase in size occurs constantly. The aging phase is characterized by a decreasing growth rate when the plant has reached maturity and and grows older (Salisbury & Ross, 1995).

Growth models belong to the category of nonlinear regression. Several types of sigmoid functions exist, including Logistic, Gompertz, Van Bertalanffy (Draper & Smith, 1998), and sigmoid model with four parameters (Widiharih et al., 2025). This study focuses on the Logistic growth model due to its simplicity, as it only involves three easily interpret parameters. The Logistic growth curve has a maximum point at M

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and one inflection point, forming a single S-shaped curve. The inflection point represents the time when the growth rate reaches its maximum value (Goshu & Koya, 2014). The Logistic growth model is a type of nonlinear regression that cannot be transformed into a linear form, making estimating its parameters more complicated. One standard method for estimating parameters is the nonlinear least square method. However, parameter estimation using the nonlinear least square method is challenging, as the parameters cannot be expressed analytically. Therefore, an iterative method is required to solve the problem. Some iteration methods for least squares optimization include Gradient Descent, Gauss Newton, and Levenberg Marquardt.

The Gradient Descent method is closely related to the learning rate. The learning rate is a constant value used by the Gradient Descent algorithm to multiply the gradient value scalarly. A small learning rate leads to a slow convergence process, while a large learning rate may cause the algorithm to overshoot the minimum, hindering convergence. On the other hand, the Gauss Newton method is based on solving a linear system, and if a singular matrix is obtained, this method will fail to solve the optimization problem so it does not find a solution. In addition, if the iteration starts from a point relatively far from the optimal point, sometimes the Gauss Newton method provides a divergent iteration (away from the optimal point). Therefore, the Levenberg Marquardt method helps solve problems in the Gradient Descent and Gauss Newton methods. The Levenberg-Marquardt method works by searching for the minimum value based on the sum of least squares, combining the speed of the Gauss-Newton method with the stability of the Gradient Descent method (Yu & Wilamowski, 2016). Thus, the method combines the best features of its predecessors while avoiding their most serious limitations (Marquardt, 1963).

Previous research conducted by Rochayani et al. (2023) showed that the Logistic growth model could provide better data predictions than the Gompertz growth model when applied to bacterial growth data. Research conducted by Bartareau et al. (2013) also showed that the Logistic growth model effectively captures the growth patterns of body length and mass in *Puma concolor coryi* (Florida Panther). This study aims to estimate the parameters of the logistic growth model for soybean plant height using the Nonlinear Least Squares method with Levenberg-Marquardt iteration. Additionally, it seeks to test the significance of the estimated parameters through a t-test to evaluate their contribution to the model. The study also examines the goodness of fit of the logistic growth model in explaining the growth pattern of soybean plant height based on the adjusted R-squared value.

2. LITERATURE REVIEW

2.1 Logistic growth model

The assumptions that must be satisfied for the Logistic growth model include tests for nonlinearity. One of the tests to detect nonlinear relationships is the Ramsey RESET (Regression Specification Error Test) test. The RESET test statistics according to Warsito and Ispriyanti (2004) and the calculation of $F_{statistic}$ can be explained as follows.

$$RESET = \frac{\frac{\hat{e}'\hat{e} - \hat{v}'\hat{v}}{p-1}}{\frac{\hat{v}'\hat{v}}{T-p}} \tag{1}$$

where b_h is the coefficient of the alternative model, $\hat{e}$ is the residual of the linear model, $\hat{e}'$ is the transpose of residual from a linear model, $\hat{v}$ is the residual of the alternative model, $\hat{v}'$ is the transpose of residual from an alternative model, p is the number of parameters in the alternative model, and T is the number of

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observations. The test statistics in the RESET test will be compared with the value of $F_{statistic}$ as the test criteria using the following formula.

$$F_{statistic} = \frac{\frac{R^2{}_v - R^2{}_u}{m}}{\frac{1 - R^2{}_v}{T - p}} \tag{2}$$

where $R^2{}_u$ is the coefficient of determination of the linear model, $R^2{}_v$ is the coefficient of determination of the alternative model, and m is the number of additional independent variables in the alternative model.

If the resulting value of RESET is greater than $F_{(k-1,n-k)}$ or p-value < α, the decision is taken to the null hypothesis failed to be rejected, meaning that the model is nonlinear. However, if the resulting value of RESET is smaller than $F_{(k-1,n-k)}$ or p-value $> \alpha$, the decision is taken that the null hypothesis is accepted, meaning that the model is linear. If the data from the model has met the assumption of nonlinearity, the data can be used in calculating the Logistic growth model. If the data from the model meets the assumption of nonlinearity, the data can be used to calculate the Logistic growth model.

Draper and Smith (1998) stated that the calculation of the Logistic growth model is based on the simplest formula for the relative growth rate, as stated in Eq. (3) and the Logistic growth model in Eq. (4).

$$\frac{dN(t)}{d(t)} = \frac{r(N(t))(M - N(t))}{M} \tag{3}$$

$$N(t) = \frac{M}{ae^{-rt} + 1} \tag{4}$$

where $N(t)$ is the plant height at time t (cm), M is the maximum plant height, r is the intrinsic growth rate, a is a parameter that controls the slope of the curve, and t is the age of the plant.

The Logistic growth curve has a maximum point at M and one inflection point, marking the transition from exponential growth to slower growth, resulting in a single S-shaped curve. The inflection point occurs under the following conditions (Gunawan, 2016).

(i) Point z is an inflection point if $N(t)$ at point z is continuous
(ii) The second derivative at point z is equal to zero ($N''(z) = 0$)
(iii) The second derivative at point t $N''(t)$ is different signs for $t < z$ and $t > z$

This is reinforced by the statement from Budimulyati et al. (2012), that determining the inflection point in the Logistic growth model can also be calculated using the following formula.

$$Inflection\ Time = \frac{ln(a)}{r} \tag{5}$$

One approach to estimate this model's parameters is using the Nonlinear Least Square (NLS) method. NLS is a form of least square analysis applied in nonlinear regression modeling, which minimizes the Residual Sum of Square (RSS) based on the parameters $\boldsymbol{\theta}$. These parameters consist of three parameters in the Logistic growth model ($M, r,$ and a), and the formula is given as follows.

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$$S(\boldsymbol{\theta}) = \sum_{t=1}^{T} \varepsilon_t{}^2(\boldsymbol{\theta}) = \left||\varepsilon_t(\boldsymbol{\theta})|\right|^2 = \varepsilon^T(\boldsymbol{\theta})\varepsilon(\boldsymbol{\theta}) \tag{6}$$

where $S(\boldsymbol{\theta})$ is Residual Sum Squares, $\varepsilon(\boldsymbol{\theta})$ is the residual, and $\varepsilon^T(\boldsymbol{\theta})$ is the transpose of the residual. The procedure in nonlinear estimation requires initial estimates of the parameters. The better the initial estimate, the faster the convergence to the optimal value. The initial value of the parameters can be found using the calculation from Table 1. In Table 1, (t_d, y_d) and (t_e, y_e) are two observation points taken with the conditions $t_e > t_d$, the indices d and e should be chosen with a sufficient distance from each other in time (t) to produce more stable parameter estimates (Draper & Smith, 1998).

Table 1. Calculation Formula for Initial Parameter Values

Parameters	**Formula**
$M^{(0)}$	y_{max}
$r^{(0)}$	$\frac{1}{t_e - t_d} ln\left(\frac{M - y_e}{M - y_d}\right)$
$a^{(0)}$	$\frac{M - y_d}{M} e^{rt_d}$

2.2 Levenberg Marquardt algorithm

The Levenberg Marquardt algorithm is an algorithm that produces a numerical solution to minimize a nonlinear function concerning the parameters in the function. The Levenberg Marquardt method combines the Gradient Descent for stability and the Gauss Newton for speed (Yu & Wilamowski, 2016).

In the Levenberg Marquardt method, regularization (λ) is added to the Hessian matrix to increase numerical stability, in the sense of making the Levenberg Marquardt method resistant to various conditions, such as when the initial guess is terrible, a singularity occurs in the Hessian matrix, or the selection of λ value of which is not correct, and so on. The formula for the Hessian matrix is as follows.

$$H_{S(LM)}(\boldsymbol{\theta}^{(k)}) \approx \left(J_f(\boldsymbol{\theta}^{(k)})\right)^T J_f(\boldsymbol{\theta}^{(k)}) + \lambda^{(k)}\mathrm{I} \tag{7}$$

The complete formula for the Levenberg-Marquardt method is:

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} - \left(\left(J_f(\boldsymbol{\theta}^{(k)})\right)^T J_f(\boldsymbol{\theta}^{(k)}) + \lambda^{(k)}\mathrm{I}\right)^{-1} \nabla S(\boldsymbol{\theta}^{(k)}) \tag{8}$$

where $\boldsymbol{\theta}^{(k+1)}$ is the updated parameter value, $\boldsymbol{\theta}^{(k)}$ is the parameter value before being updated, $\lambda^{(k)}$ is the regularization for each iteration, and $\nabla S(\boldsymbol{\theta}^k)$ is the gradient vector at k iteration.

The λ is a regularization value that must not be negative and will be adjusted at each iteration. If the value of λ is small (approaching 0), this method will approach the Gauss Newton method, which will result in five small error results. When the value of λ is large, this method will approach the Gradient Descent method. The λ is typically adjusted adaptively. If the improvement in iteration is small, the λ is increased, and if the improvement is significant, the λ is decreased. In summary, the steps in the Levenberg Marquardt algorithm are as follows:

1. Initialize the initial parameter values using the provisions listed in Table 1.

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2. Set the tolerance limit under the following conditions:
   (i) $\left|\boldsymbol{\theta}^{(k+1)} - \boldsymbol{\theta}^{(k)}\right| < 10^{-6}$
   (ii) $\left|S\left(\boldsymbol{\theta}^{(k+1)}\right) - S\left(\boldsymbol{\theta}^{(k)}\right)\right| < 10^{-6}$
   (iii) Maximum iterations are 50
3. Calculating Residual Sum Squares ($S(\boldsymbol{\theta})$).

$$S(\boldsymbol{\theta}) = \sum_{t=1}^{j} (\varepsilon_t(\boldsymbol{\theta}))^2$$

4. Constructing the Jacobian matrix ($J_f(\boldsymbol{\theta})$).

$$J_f(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial \varepsilon_1(\boldsymbol{\theta})}{\partial \theta_1} & \cdots & \frac{\partial \varepsilon_1(\boldsymbol{\theta})}{\partial \theta_p} \\ \vdots & \ddots & \vdots \\ \frac{\partial \varepsilon_j(\boldsymbol{\theta})}{\partial \theta_1} & \cdots & \frac{\partial \varepsilon_j(\boldsymbol{\theta})}{\partial \theta_p} \end{bmatrix}$$

by using the residual formula:

$$\varepsilon_t\left(\boldsymbol{\theta}^{(k)}\right) = y_t - \frac{M}{ae^{-rt} + 1}$$

the derivative formula for each parameter is obtained as follows.

Table 2. Residual derivatives with respect to parameters

Parameters	**Derivative Results**
M	$\frac{\partial \varepsilon_t}{\partial M}\left(\frac{M}{ae^{-rt}+1}\right) = \frac{1}{ae^{-rt}+1}$
r	$\frac{\partial \varepsilon_t}{\partial r}\left(\frac{M}{ae^{-rt}+1}\right) = \frac{Mate^{-rt}}{(ae^{-rt}+1)^2}$
a	$\frac{\partial \varepsilon_t}{\partial r}\left(\frac{M}{ae^{-rt}+1}\right) = \frac{Mate^{-rt}}{(ae^{-rt}+1)^2}$

Therefore, if the three derivatives are combined in a matrix, the resulting Jacobian matrix will be as follows.

$$J\left(\boldsymbol{\theta}^{(k)}\right) = \begin{bmatrix} \frac{\partial \varepsilon_1}{\partial M} & \frac{\partial \varepsilon_1}{\partial r} & \frac{\partial \varepsilon_1}{\partial a} \\ \frac{\partial \varepsilon_2}{\partial M} & \frac{\partial \varepsilon_2}{\partial r} & \frac{\partial \varepsilon_2}{\partial a} \\ \frac{\partial \varepsilon_3}{\partial M} & \frac{\partial \varepsilon_3}{\partial r} & \frac{\partial \varepsilon_3}{\partial a} \\ \vdots & \vdots & \vdots \\ \frac{\partial \varepsilon_j}{\partial M} & \frac{\partial \varepsilon_j}{\partial r} & \frac{\partial \varepsilon_j}{\partial a} \end{bmatrix} = \begin{bmatrix} \frac{1}{ae^{-r}+1} & \frac{Mae^{-r}}{(ae^{-r}+1)^2} & \frac{-Me^{-2r}}{(ae^{-2r}+1)^2} \\ \frac{1}{ae^{-2r}+1} & \frac{2Mae^{-2r}}{(ae^{-2r}+1)^2} & \frac{-Me^{-2r}}{(ae^{-2r}+1)^2} \\ \frac{1}{ae^{-3r}+1} & \frac{3Mae^{-3r}}{(ae^{-3r}+1)^2} & \frac{Me^{-3r}}{(ae^{-3r}+1)^2} \\ \vdots & \vdots & \vdots \\ \frac{1}{ae^{-jr}+1} & \frac{jMae^{-jr}}{(ae^{-jr}+1)^2} & \frac{-Me^{-jr}}{(ae^{jtr}+1)^2} \end{bmatrix} \tag{9}$$

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where M is the maximum plant height, r is the intrinsic growth height, and a is the controller of the slope of the Logistic growth curve.

5. Calculating the gradient of Residual Sum Squares ($\nabla S(\boldsymbol{\theta})$).

   $\nabla S(\boldsymbol{\theta}) \cong \left(J_f(\boldsymbol{\theta})^T\right)\varepsilon(\boldsymbol{\theta})$

   where $\varepsilon(\boldsymbol{\theta})$ is the residual vector $J_f(\boldsymbol{\theta})^T$ is the transpose of the Jacobian matrix

6. Construct the Hessian matrix ($H_{S(LM)}(\boldsymbol{\theta})$) for the Levenberg Marquardt method.

   $H_{S(LM)}\left(\boldsymbol{\theta}^{(k)}\right) \approx \left(J_f\left(\boldsymbol{\theta}^{(k)}\right)\right)^T J_f\left(\boldsymbol{\theta}^{(k)}\right) + \lambda^{(k)}\mathrm{I}$

7. Calculate the parameter value in the next iteration ($\boldsymbol{\theta}^{(k+1)}$).

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} - \left(\left(J_f\left(\boldsymbol{\theta}^{(k)}\right)\right)^T J_f\left(\boldsymbol{\theta}^{(k)}\right) + \lambda^{(k)} I\right)^{-1} \nabla S\left(\boldsymbol{\theta}^{(k)}\right) \tag{10}$$

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} - \left(H_{S(GN)}\left(\boldsymbol{\theta}^{(k)}\right) + \lambda^{(k)}\mathrm{I}\right)^{-1} \nabla S\left(\boldsymbol{\theta}^{(k)}\right) \tag{11}$$

   where $\boldsymbol{\theta}^{(k)}$ is the current parameter value, $\boldsymbol{\theta}^{(k+1)}$ is the parameter value in the next iteration, and $\boldsymbol{\delta}^{(k)}$ is the value that will be added to $\boldsymbol{\theta}^{(k)}$ to obtain the next iteration.

8. Repeat the previous steps until the iteration reaches one of the convergence limits.

At the sixth step of the Levenberg Marquardt method, a correlation was obtained between Gauss Newton, Gradient Descent, and Levenberg Marquardt. Sunarauw (2018) stated when the value of λ is very small ($\lambda \rightarrow 0$), the update rule of the Levenberg Marquardt algorithm in Eq. (10) becomes similar to the Gauss Newton algorithm. Conversely, when the value of λ is very large ($\lambda \rightarrow \infty$), the Levenberg Marquardt algorithm update becomes similar to the Gradient Descent algorithm, so that there is a relationship between λ and α.

$$\alpha = \frac{1}{\lambda} \tag{12}$$

When the iteration is convergent, the best parameters are obtained using the Logistic growth model.

2.3 Significance parameter test

Parameters obtained from the numerical algorithm must be tested for parameter significance to see whether the parameters have a significant effect on the model using the t-test. The test statistic used are listed in Table 3 with the condition of the parameter $M, r, a \in R^+$.

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Table 3. Test of the t-Statistic

Parameters	Statistic
M	$t = \frac{\hat{M}}{SE(\hat{M})}$
r	$t = \frac{\hat{r}}{SE(\hat{r})}$
a	$t = \frac{\hat{a}}{SE(\hat{a})}$

The t-test of significance requires the calculation of the standard error of the estimator. This standard error calculation uses Fisher information, which will estimate of how confident we are in the parameter estimate $\boldsymbol{\theta}$ produced (Tae, 2020). The formula for finding the Fisher value in the Nonlinear Least Square method is as follows (Seber & Wild, 2005).

$$I(\boldsymbol{\theta}) = -\left(H(\boldsymbol{\theta})\right)^{-1} \tag{13}$$

with the contents of the Fisher matrix consisting of:

$$I(\boldsymbol{\theta}) = \begin{bmatrix} I(M,M) & I(M,r) & I(M,a) \\ I(r,M) & I(r,r) & I(r,a) \\ I(a,M) & I(a,r) & I(a,a) \end{bmatrix}$$

$SE(\hat{\boldsymbol{\theta}})$ or standard error of each parameter is the root of the variance of each parameter so that $SE(\hat{\boldsymbol{\theta}})$ for each parameter can be calculated by taking the diagonal root of $I(\boldsymbol{\theta})$.

$$SE(\hat{M}) = \sqrt{I(M,M)^{-1}} \tag{14}$$

$$SE(\hat{r}) = \sqrt{I(r,r)^{-1}} \tag{15}$$

$$SE(\hat{a}) = \sqrt{I(a,a)^{-1}} \tag{16}$$

The test criteria are to reject the null hypothesis if the value of t that results from the t-test is greater than t_{table} or the p-value is less than α, meaning that the test parameter significant influences the model. The null hypothesis failed to be rejected if the value of t is less than or equal to t_{table}, so it can be concluded that the parameters tested do not significant affect the model.

The next step is to perform the classical assumption test for the nonlinear model. The assumptions of the nonlinear model are used to ensure that the obtained model is appropriate and usable. The nonlinear model must satisfy the following assumptions (Ritz & Streibig, 2008).

(i) Residuals follow a normal
The normality of residuals can be evaluated using the Shapiro Wilk test. The formula for the Shapiro-Wilk test is as follows (Shapiro & Wilk, 1965).

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$$W = \frac{\left(\sum_{i=1}^{n} a_i Y_{(i)}\right)^2}{\sum_{i=1}^{n} (Y_i - \bar{Y})^2} \tag{17}$$

where $Y_{(i)}$ is the ordered data sequence, a_i is a constant that depends on the sample size, and $\bar{Y}$ is the mean of the sample data.

(ii) Independence of residuals ε_t
The autocorrelation test is used to examine the independence of residuals, specifically to determine whether there is a correlation between the residual at time t and $t-1$ in a regression model. The autocorrelation test can be conducted using the Durbin-Watson test. The test statistic used is:

$$d = \frac{\sum_{i=1}^{n} (e_i - e_{i-1})^2}{\sum_{i=1}^{n} {e_i}^2} \tag{18}$$

The test criteria are based on the values of d_L (Lower Critical Value) and d_U (Upper Critical Value). The following Table 4 presents the decision rule based on the values of d_L and d_U.

Table 4. Decision Rule for the Durbin Watson Test

Conditions	Decision	Explanation
$d < d_L$ or $d > 4 - d_L$	Reject H_0	Autocorrelation
$d_L \le d \le d_U$ or $4 - d_U \le d \le 4 - d_L$	No decision	No conclusion can be drawn
$d_L \le d \le 4 - d_U$	Fail to reject H_0	No autocorrelation

(iii) Homoscedasticity of variance
Homoscedasticity can be tested in various ways, one of which is the White test. The formulation of the White test is as follows (White, 1980).

$$nR^2 \approx \chi^2 \tag{19}$$

Data prediction is carried out using parameters that have successfully passed the t-test and classical assumption test. When the data prediction has been obtained, it is continued with the goodness of the model using the goodness metric, namely adjusted R-squared (${R^2}_{adj}$). Adjusted R-squared is used to address the limitation of R-squared, which tends to increase with the addition of new parameters, even if those parameters are not relevant (Grekousis, 2020). The formula for adjusted R-squared is as follows.

$${R^2}_{adj} = 1 - \left(\frac{(1 - R^2)(j - 1)}{j - p - 1}\right) \tag{20}$$

with R^2 is the R-squared calculation value, j is the number of observations, and p is the number of parameters used.

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3. METHODOLOGY

The type of data used is secondary data, namely data taken from the average height of 26 soybean plants in the observation of the 2023 food crop production technology practicum, Agroecotechnology study program, Diponegoro University from September 20 to December 22, 2023 (10 weeks). The type of soybean variety is the Grobogan variety and the treatment given is the type of fertilizer with a dose of 0 kg/ha P_2O_5 (the absence of phosphate fertilizer (P_2O_5)) and planted in the Agroecotechnology practicum planting land. The process of this research involves the following steps:

1. Setting the initial values of the parameter $\boldsymbol{\theta}^{(k)}$, including $M, r,$ and $a,$ in the manner stated in Table 1 and the iteration index (k) starts from 0.
2. Implementing the Levenberg Marquardt algorithm as described in Section 2.4
3. Calculating the parameter significance test using the t-test to see the significance of the parameters to the model.
4. Calculate and interpret the results of the goodness of fit of the model using adjusted R-squared.

4. RESULTS AND DISCUSSION

The nonlinearity assumption test can be done by looking at the data visualization. The curve shape of the nonlinear model cannot be described with a straight line. The data visualization is shown in Figure 1. The visualization in Figure 1 does not prove the nonlinear pattern enough, so a statistical assumption test is carried out. This assumption test is carried out to see whether the data used is nonlinear or not by looking at the RESET value obtained. RESET value is 8.276901, bigger than F_{table}= 5.317655, so it can be concluded that the data between the age of soybean plants and the age of plants has a nonlinear relationship. Therefore, the model that will be used is nonlinear.

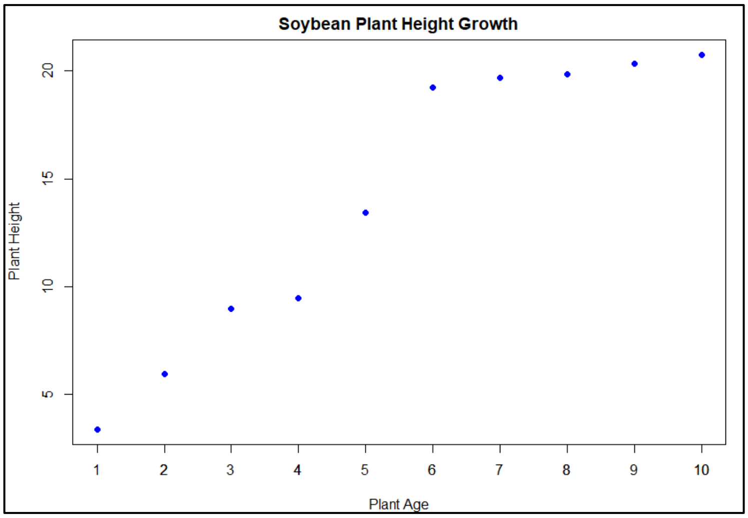

Fig 1. Scatter diagram of plant age with plant height

The initial parameters of the Levenberg Marquardt are listed in Table 5. The computation was conducted using the ‘nlsLM’ package in RStudio. After 13 iterations, the best parameters were obtained, as shown in Table 6.

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Table 5. Initial parameter value results

Parameters	**Initial Value**
$\boldsymbol{M}$	20.75
$\boldsymbol{r}$	−0.46655
$\boldsymbol{a}$	0.28035

Table 6. Parameter estimation results

Parameters	**Estimate**
$\boldsymbol{M}$	21.6543
$\boldsymbol{r}$	0.61368
$\boldsymbol{a}$	10.5104

The logistic growth model obtained from the parameter estimates produces the following model.

$$N(t) = \frac{21.6543}{10.5104e^{-0.61368t} + 1} \tag{21}$$

Based on Eq. (21), the obtained logistic model describes the growth pattern of soybean plant height, which follows a sigmoid curve. The estimated parameter values indicate that the maximum attainable plant height is 21.6543 cm, with an intrinsic growth rate of 0.61368, and a slope-controlling parameter of 10.5104. The inflection point is calculated as follows:

$$Inflection\ point = \frac{ln(a)}{r} = \frac{ln(10.5109)}{0.61371} = 3.833102$$

There is a sign change from positive to negative around the inflection point which is described in Table 7.

Table 7. Second derivative value around the inflection point 3.833102

Condition	**Point *s***	$\mathbf{N''(t)}$	**Information**
s < inflection point	3.833101	6.210922×10^{-7}	> Zero (0)
s > inflection point	3.833103	-6.30226×10^{-7}	< Zero (0)

The calculation results show that the inflection point occurs at t = 3.833102 weeks. Thus, it can be concluded that at approximately 3.8 weeks of age, soybean plants achieve their maximum growth rate, after which growth continues but at a decreasing rate until approaching the maximum height of 21.6543 cm.

A parameter significance test was conducted using a t-test. The results of the t-test for each parameter are presented in Table 8. The calculations in Table 8 give the result that it is rejected for all parameters because the calculated t statistic for parameters $M, r,$ and a > t_{table} (1.8946). Therefore, all parameters in the soybean plant growth model significantly affect or contribute to the model.

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Table 8. Results of t-test calculation

Parameters	Parameter Estimation	SE ($\hat{\theta}$)	t Statistic
M	21.6543	1.3517	16.0191
r	0.61368	0.1219	5.0326
a	10.5104	4.4389	2.3678

The logistic sigmoid growth model on plant age data against soybean plant height involves three assumption tests: normality, heteroscedasticity, and autocorrelation. The calculation of all classical assumption tests uses assistance from RStudio. The normality test uses Shapiro-Wilk because the data available is very little (less than 30 data). The p-value is 0.9617 > α = 5%, which indicates that the residual data on soybean plant height is normally distributed. Furthermore, the heteroscedasticity test produces a p-value is 0.7605 > α = 5%, indicating that the homoscedasticity assumption is met. In addition to the normality and homoscedasticity tests, there is an autocorrelation test that needs to be checked. The Durbin-Watson value is 1.8937, within the range:

$$d_L(0.5253) < DW\ (1.8927)\ <\ 4\ -\ d_U\ (4\ -\ 2.0163\ =\ 2.6803)$$

suggesting that the residual data on soybean plant height does not exhibit autocorrelation.

The sigmoid requirements, parameter significance, and assumption test have all been satisfied, indicating that the three parameters are appropriate for use in the model to predict the data. Table 8 summarizes the calculation results of the prediction using the Logistic growth function.

Table 8. Data prediction results

Plant Age (Week)	Result (cm)
1	3.23680
2	5.30709
3	8.11789
4	11.38094
5	14.54613
6	17.12426
7	18.94165
8	20.09622
9	20.78196
10	21.17308

The results given in Table 8 will calculate the model's goodness of fit metric using adjusted R-squared. The adjusted R-squared value obtained using Eq. (22) of 0.957113 has the interpretation that the model is able to explain 95.71% of the variation in the actual data after considering the number of parameters used. This value is considered high and matches with the actual data. Figure 2 provides a visualization of the curve formed from the results of the Logistic growth model based on Table 8.

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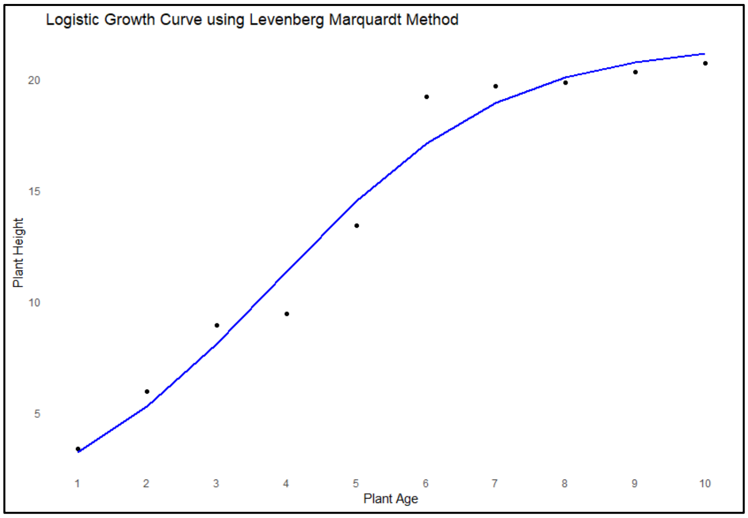

Fig 2. Curve generated from prediction data

5. CONCLUSION AND FUTURE DIRECTIONS

Based on the results of the research conducted, the data used meets the nonlinearity assumption test. The Levenberg Marquardt approach results in three parameter estimates, the maximum height (M) was 21.65 cm, the intrinsic growth rate (r) was 0.61 cm/week, and the parameter controlling the slope of the curve (a) was 10.51. These parameters produce the following growth model.

$$N(t) = \frac{21.65}{10.51e^{-0.61t} + 1}$$

The parameter estimates from the Levenberg Marquardt iteration passed the significance test using the t-test, meaning the parameters in the soybean plant height growth model have a significant impact or contribute to the model. Furthermore, the adjusted R-squared value obtained is 0.957113, which indicates that the model is able to explain 95.71% of the variation in the actual data after considering the number of parameters used. This model has performed very well in estimating the height of soybean plants used in this study.

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7. CONFLICT OF INTEREST STATEMENT

The authors declare that this research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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8. AUTHORS' CONTRIBUTIONS

Nathifa Fadheela: Data collection, analyzing, and drafting the original manuscript. **Agus Rusgiyono:** Conceptualizing, developing methodology, and analyzing. **Masithoh Yessi Rochayani:** Analyzing, drafting the original manuscript, and editing.

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