

# Development of the Spherical First Order Polarization Tensor Calculator

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## ABSTRACT

The polarization tensor is implemented in some applications, especially in science and engineering, to identify or characterize objects. An object can be represented as its polarization tensor, and an ellipsoid can have the same polarization tensor as the object. Thus, investigating an ellipsoid with its respective polarization tensor might provide additional information about the real object. Motivated by this research, this study aims to develop a standalone application using a graphical user interface (GUI) in MATLAB, called the Spherical First Order Polarization Tensor Calculator to enhance computation efficiency related to the first order PT for a sphere. This application consists of three main features, which are to calculate the first order PT for sphere, to determine the sphere's radius, and to illustrate the sphere in a three-dimensional graph. A brief demonstration of each function is provided, and the application's reliability is also validated. The findings suggest a quicker approach for computing the first order PT related to the sphere, which can serve as a reference for other researchers in the related area.

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## 1. INTRODUCTION

The study of polarization tensor (PT) has a crucial role in science and engineering field. It is widely applied in electrical, electromagnetic, and magnetic fields. The application of PT includes metal detection for security screening and landmine clearance (Dekdouk et al., 2014; Marsh et al., 2013). Additionally, PT has been adapted biologically to investigate electro-sensing fish in characterizing and discriminating objects (Khairuddin, 2016). Moreover, in the most recent study, PT is used for the detection of vapes and batteries within an electro-magnetic field using a coil system (Williams et al., 2024). In these applications, PT helps

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to extract important information about conducting object, including its material characteristics, shape and orientation (Ammari & Kang, 2007).

The most basic form of PT is the first order PT. The formula of the first order PT for ellipsoid was originally given by Ammari and Kang (2007). It was modified by Yunos and Khairuddin (2017) through the implementation of depolarization factors into the original formula. As demonstrated by Khairuddin and Lionheart (2013), the PT for various objects can be expressed through the first order PT for ellipsoids, yet there is still significant interest in studying the first order PT for spheres.

A sphere can be represented by Eq. (1) in Cartesian coordinate system given by

$$\frac{x^2+y^2+z^2}{r^2} = 1, \quad (1)$$

where  $r$  is the radius of the sphere.

Due to the high interest in the PT, many numerical methods have been used by previous researchers in determining the first order PT (such as in Capdeboscq et al., 2012; Lu et al., 2015; Sukri et al., 2021; Sukri et al., 2025) and some research did focus on a sphere as the geometry (such as Sukri et al., 2020). In order to increase the efficiency of calculating the first order PT, a standalone application called Spheroidal First Order PT was developed by Yunos et al. (2023). This application was initially designed to calculate the spheroidal first order PT, mainly for spheroids with certain conditions. Users are required to insert the spheroid's conductivity,  $k$  along with its semi axes  $a$  and  $b$ . In addition, this application has a function to estimate the spheroid based on the its PT by fixing the conductivity value. Subsequently, Yunos et al. (2024) upgraded the existing application by incorporating new functions. These upgrades include the computation for all types of spheroids. However, these applications were not designed to handle the spherical case, resulting in an important limitation: the first order PT of spheres cannot be computed, analysed, or compared using existing applications.

Therefore, an application named the Spherical First Order Polarization Tensor Calculator is developed to provide easy computation related to the first order PT for sphere. Hence, the objective of this study is to extend the invention of the first order PT calculator for spheres. The functions of the application consist of calculating the first order PT for sphere given that the conductivity and radius are known, computing the radius of sphere given that the conductivity and the PT of the sphere are known as well as visualizing the sphere in three-dimensional space based on its radius.

## 2. MATHEMATICAL FORMULATION OF THE FIRST ORDER POLARIZATION TENSOR OF A SPHERE

This section reviews the formula of the first order PT, where the analytical solution of the PT for ellipsoid has been described by Ammari and Kang (2007). The formula was then modified by Yunos and Khairuddin (2017) through the implementation of depolarization factors into the formula. The formula can be represented as a  $3 \times 3$  diagonal matrix, where the diagonal elements are real number.

Assume the object,  $B$  is an ellipsoid, the first order PT denoted as  $M(k, B)$  is given by

$$M(k, B) = (k - 1)|B| \begin{bmatrix} \frac{1}{(1-d_1)+kd_1} & 0 & 0 \\ 0 & \frac{1}{(1-d_2)+kd_2} & 0 \\ 0 & 0 & \frac{1}{(1-d_3)+kd_3} \end{bmatrix} \quad (2)$$

where  $k$  is the conductivity of the ellipsoid, while  $|B|$  is the volume of the ellipsoid and  $d_i$  for  $i = 1, 2, 3$  are the depolarization factors. These depolarization factors with semi axes  $a$ ,  $b$  and  $c$  are given by Milton (2002) and defined as in Eq. (3), Eq. (4) and Eq. (5).

$$d_1 = \frac{abc}{2} \int_0^\infty \frac{1}{(a^2+y)^{\frac{3}{2}} \sqrt{(b^2+y)(c^2+y)}} dy, \quad (3)$$

$$d_2 = \frac{abc}{2} \int_0^\infty \frac{1}{(b^2+y)^{\frac{3}{2}} \sqrt{(a^2+y)(c^2+y)}} dy, \quad (4)$$

$$d_3 = \frac{abc}{2} \int_0^\infty \frac{1}{(c^2+y)^{\frac{3}{2}} \sqrt{(a^2+y)(b^2+y)}} dy, \quad (5)$$

However, when  $B$  is a sphere, such that it has the same axes that is  $a = b = c$ , the depolarization factors,  $d_1$ ,  $d_2$ , and  $d_3$  will have equivalent value which is  $\frac{1}{3}$ . As a result, the first order PT in Eq. (2) can be simplified to

$$M(k, B) = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_1 \end{bmatrix}, \quad (6)$$

where

$$M_1 = \frac{3|B|(k-1)}{2+k}, \quad (7)$$

such that  $|B| = \frac{4}{3}\pi r^3$ , where the semi axis is denoted as the radius of the sphere,  $r$ . Hence, the first order PT of sphere can be computed by using Eq. (6) and Eq. (7). Fig. 1 presents the procedure for calculating the first order PT of a sphere,  $M(k, B)$ , given fixed values of  $k$  and  $r$ .

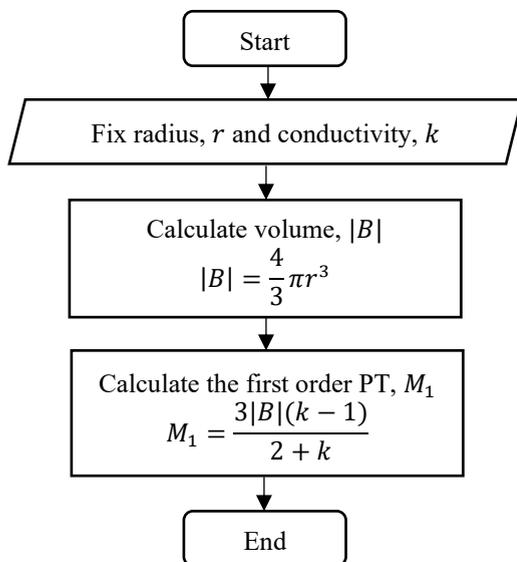


Fig. 1. Flowchart for computing the first order PT of a sphere, at a fixed radius and conductivity

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Next, to determine the radius of the sphere, Eq. (7) need to be modified by finding the volume first based on the first order PT and then determining the radius. The derivations are given as follows,

$$|B| = \frac{M_1(2+k)}{3(k-1)}, \quad (8)$$

$$r = \sqrt[3]{\frac{3|B|}{4\pi}}. \quad (9)$$

In order to calculate the radius of the sphere in Eq. (9), the given first order PT is constrained to follow the matrix in Eq. (6), with the condition that  $M_1 > 0$  or  $M_1 < 0$ . Thus, based on these conditions, the conductivity can be set to specific values according to the findings by Khairuddin et al. (2017), where a positive definite PT should have a conductivity greater than 1, while a negative definite PT should have a conductivity less than 1 but still positive. Fig. 2 illustrates the flowchart of computing the radius of the sphere.

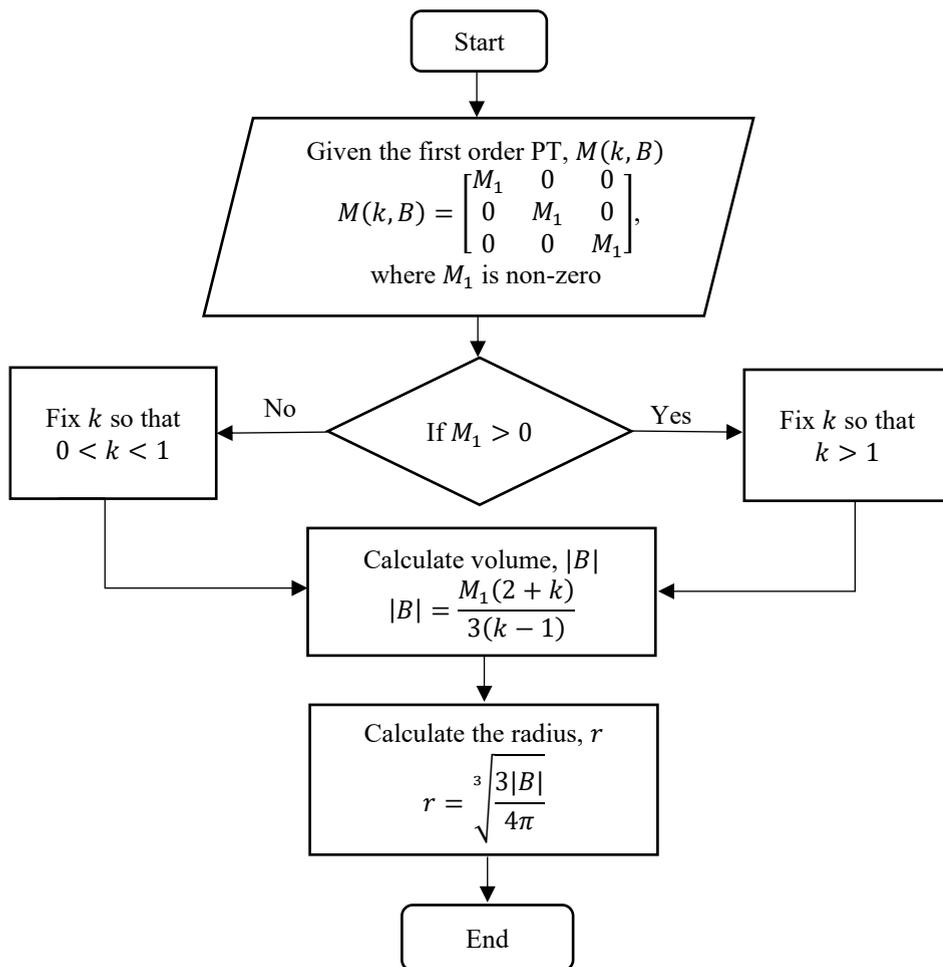


Fig. 2. Flowchart of computing the radius,  $r$  from the given first order PT

Hence, the application will implement the algorithms as demonstrated in Fig. 1 and Fig. 2 for computing the spherical first order PT given that the conductivity and radius are known and also for computing the radius given that the PT and the conductivity are known.

### 3. METHODOLOGY

The concept of graphical user interface (GUI) has progressed significantly, providing a user-friendly platform for an easy way of engaging with the computational tools. MATLAB, a leading software for mathematical computation, assists this advancement through its App Designer, an integrated platform that allows users to design and code interactive applications. App Designer provides a set of pre-built user interface (UI) components which can be easily dragged and arranged into desired design. Additionally, it offers a code view where developers can program these components, enabling seamless integration of user inputs with underlying computational algorithms. For instance, the study from Yunos et al. (2024) developed a GUI for computing the spheroidal first order PT utilizing MATLAB App Designer. The toolkit accelerates complex numerical computations of spheroidal first order PT.

The initial step of developing the Spherical First Order Polarization Tensor Calculator is to design the interface of the desired layout of the application in GUI. The layout is designed suitable for the objectives of this development of application. Next, the codes involved for the computation of the spherical first order PT are programmed. The visualisation of the three-dimensional sphere is also given. Fig. 3 presents the process of developing the Spherical First Order Polarization Tensor Calculator application

#### 3.1 Generalization of graphical user interface (GUI)

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Based on the algorithm outlined in Fig. 1, the first order PT of a sphere,  $M(k, B)$  can be computed when its conductivity,  $k$  and radius,  $r$  are specified. Therefore, users need to input the values of  $k$  and  $r$  into the application. In order to compute the radius of the sphere, the required inputs are the first order PT,  $M_1$  and the conductivity,  $k$  of the sphere as shown in Fig. 2. Next, the input required to visualize the sphere in three-dimensional space is the radius,  $r$  of the sphere.

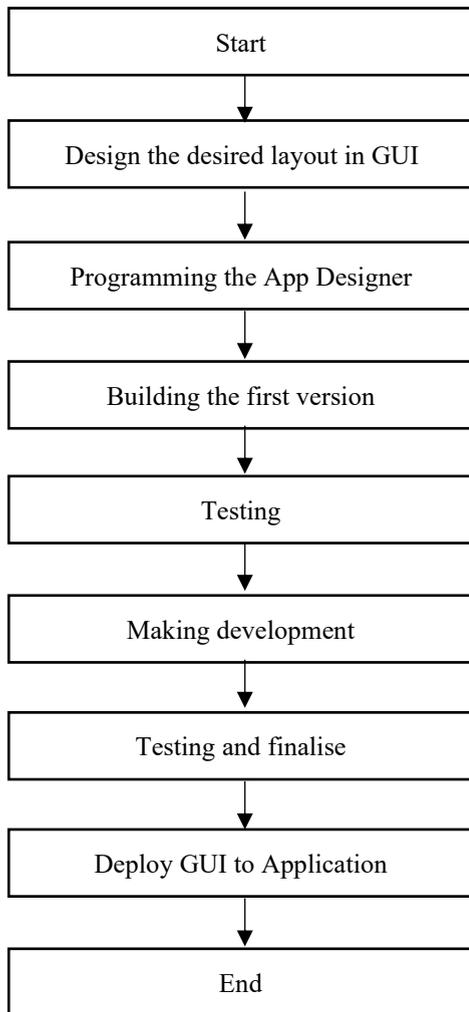


Fig. 3. The flowchart of the process of developing the Spherical First Order Polarization Tensor Calculator application

### 3.2 Features in graphical user interface (GUI)

This application is developed by using the App Designer in MATLAB. The App Designer can be accessed by typing “appdesigner” in the Command Window. Once launched, the App Designer window opens, presenting the options to start a new application. From the App Designer page, the Blank App template is selected to begin developing a new GUI application. A design workspace will appear with a blank canvas and Component Palette on the right-hand side (see Fig. 4).

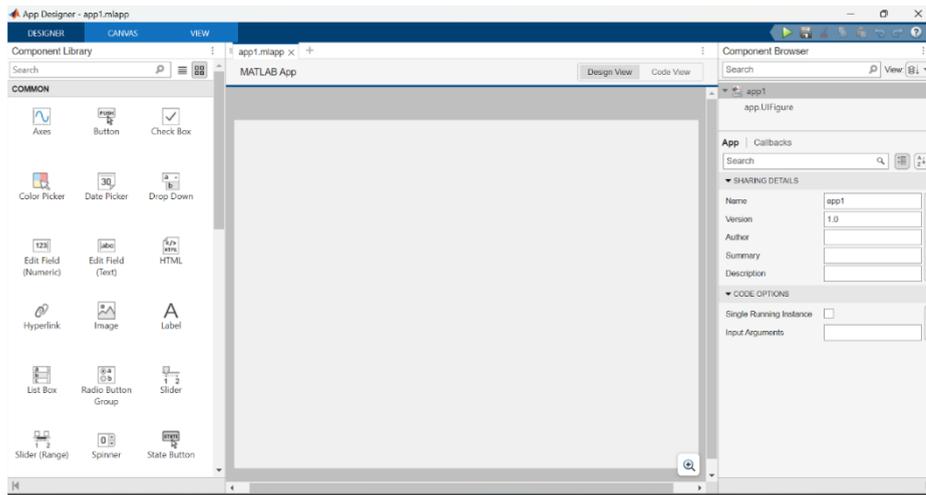


Fig. 4. The design workspace with a blank canvas and Component Palette

Then, suitable components are selected for the design of the application. In this case, button, edit field (numeric), edit field (text), text area, panel and axes are selected. These components are dragged and dropped onto the canvas and arranged according to the desired application design. After the design is completed, the next step is to write the code that defines the functionality of the components. From the Code View tab, the automatically generated callbacks are used to implement the component functions. For example, the formulas of the volume of the sphere,  $|B|$ , the radius of the sphere,  $r$  and the first order PT of sphere,  $M_1$  (shown in Fig. 5).

Once the coding is done, the GUI is tested by pressing the Run button located at the top of the App Designer toolbar. This action opens up the app in a standalone window. The components behavior is verified and any issues in the code are adjusted. Once the GUI is complete and functional, the app is saved as an .mlapp file. The application is then converted into a standalone application that can be installed and accessed by both MATLAB or non-MATLAB users.

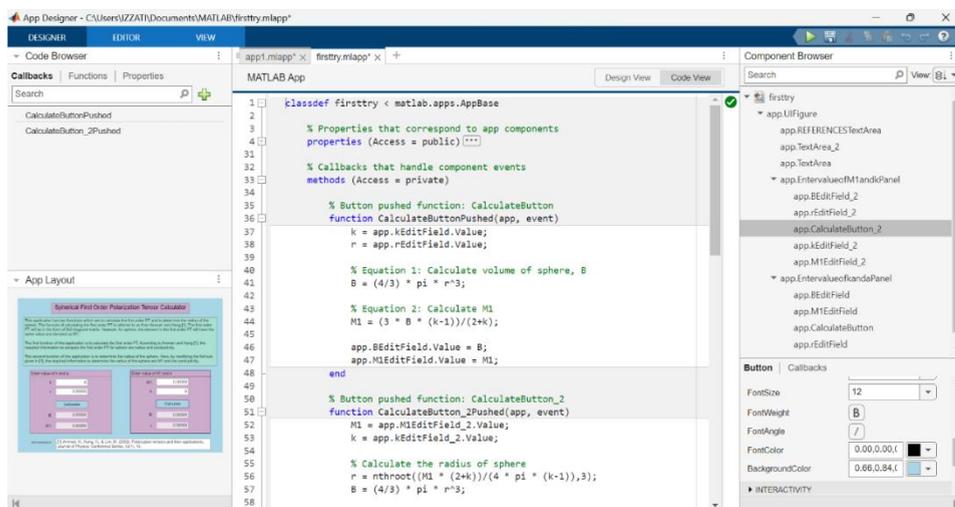


Fig. 5. The Code View in App Designer

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## 4. RESULTS

In this section, the graphical user interface (GUI) for the Spherical First Order Polarization Tensor Calculator will be presented and demonstrated, design to assist three key calculations for sphere.

### 4.1 Interface view of spherical first order polarization tensor calculator

The GUI comprises of three main functions:

- (i) to calculate the first order PT for sphere
- (ii) to calculate the radius of sphere
- (iii) to visualize the sphere in three-dimensional space.

The framework code for the GUI is designed to implement graphical controls in specific positions within the interface. It contains various callbacks, which hold all the additional functions related to the GUI controls.

Fig. 6 displays the complete interface of the Spherical First Order Polarization Tensor Calculator developed in MATLAB App Designer. The interface view begins with a brief description about the application's functionality. Then, each function is separated into distinct sections using the panel component for a neat and user-friendly design.

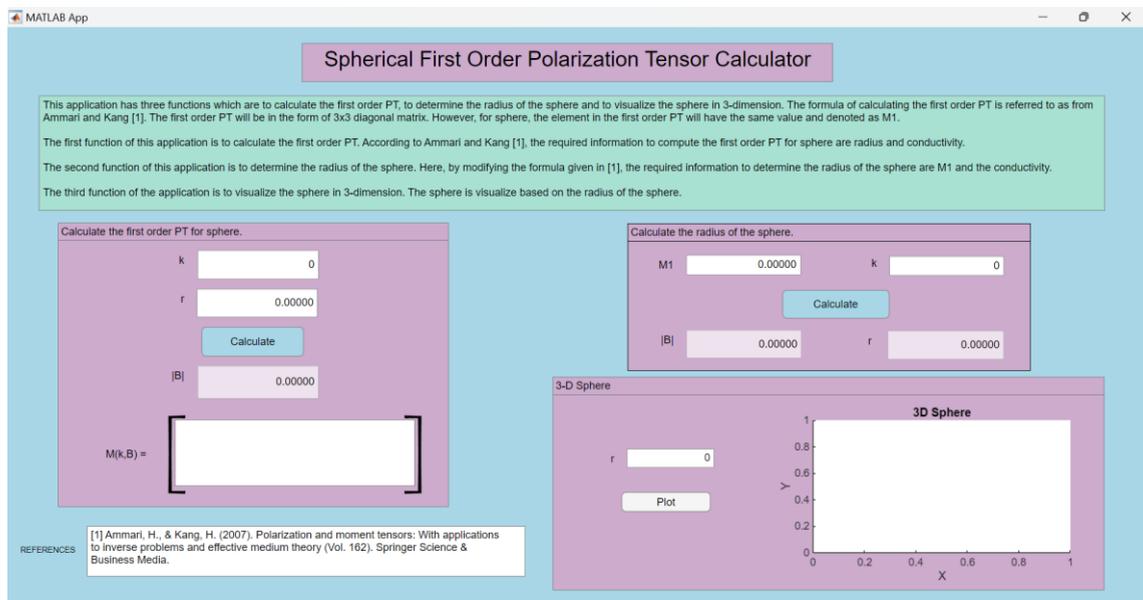


Fig. 6. The interface view of the Spherical First Order Polarization Tensor Calculator

### 4.2 Calculate the first order PT for a sphere

The first feature of the application is to compute the first order PT for a sphere. Its core script executes the calculation following the procedure shown in Fig. 1. Fig. 7 shows the interface view for calculating the first order PT for sphere before the calculation is executed. The values of conductivity,  $k$  and radius,  $r$  must be entered in the designated fields. Once the inputs are inserted, the 'Calculate' button should be clicked to execute the programmed calculations. When the 'Calculate' button is clicked, the resulting values, which

are the volume of the sphere,  $|B|$  and the first order PT of the sphere will promptly display in the  $|B|$  and  $M(k, B)$  box.

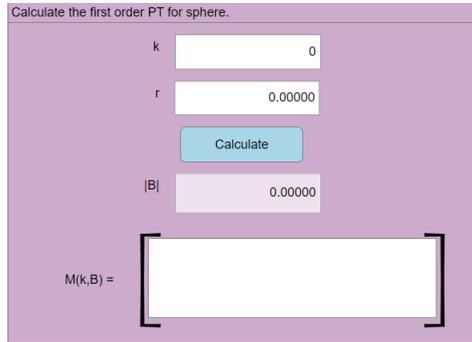


Fig. 7. The interface view of calculating the first order PT for sphere before calculation is executed

In order to demonstrate the reliability of the application, the first order PT is calculated by considering the values of  $k = 10$  and  $r = 1$ . Fig. 8 shows the results obtained.

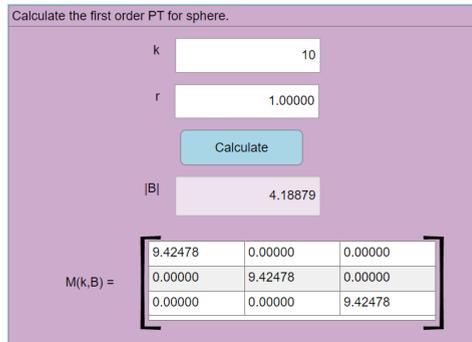


Fig. 8. The interface view of calculating the first order PT for sphere after calculation is executed

Fig. 8 illustrates that for  $k = 10$  and  $r = 1$ , the corresponding values are calculated as  $|B| = 4.18879$  and

$$M(k, B) = \begin{bmatrix} 9.42478 & 0 & 0 \\ 0 & 9.42478 & 0 \\ 0 & 0 & 9.42478 \end{bmatrix}.$$

### 4.3 Calculate the radius of a sphere

The second function of the application is to calculate the radius of the sphere,  $r$ . The primary code is executed based on the algorithm in Fig. 2. Fig. 9 shows the layout for calculating the radius of the sphere before calculation is executed. The values of the first order PT, specifically the diagonal element of  $M(k, B)$  denoted as  $M_1$  and the conductivity,  $k$  must be entered in the designated fields. After the required inputs have been provided, clicking the ‘Calculate’ button will initiate the built-in computation. Once clicked, the application will promptly display the output values which are the volume of the sphere,  $|B|$  and the radius of the sphere in the  $|B|$  and  $r$  box.

Fig. 9. The layout of calculating the radius of the sphere before calculation is executed

In order to demonstrate the reliability of the second function of this application, the radius is calculated by considering the values of  $M_1 = 9.42478$  and  $k = 10$ . This example is taken from the output in Fig. 8. Fig. 10 shows the results obtained.

Fig. 10. The interface view of calculating the radius of the sphere after the calculation is executed

Fig. 10 illustrates that for  $M_1 = 9.42478$  and  $k = 10$ , the corresponding values are calculated as  $|B| = 4.18879$  and  $r = 1$ . It is shown that the values of volume,  $|B|$  and the radius,  $r$  obtained are exactly the same as the input and output in Fig. 8, which suggests the errors for both  $|B|$  and  $r$  are zero. Therefore, it can be concluded that the developed application is valid.

#### 4.4 Plot three-dimensional sphere

The last function of the application is visualizing the sphere in three dimensions. The sphere is visualized based on its radius. Fig. 11 shows the layout for visualizing the sphere in three-dimensional space before execution. The radius of the sphere,  $r$  must be entered in the designated field. Once this input is provided, clicking the 'Plot' button will execute the programmed coding. The output comprising the three-dimensional sphere will promptly appear in the graph.

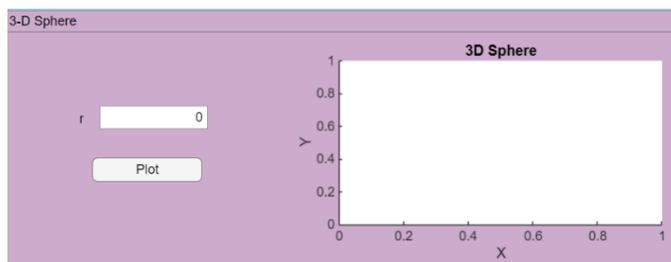


Fig. 11. The interface view for visualizing the sphere in three-dimensional space before execution

In order to demonstrate the reliability of the third function of this application, the radius of the sphere is considered as  $r = 5$ . Fig. 12 shows the result obtained.

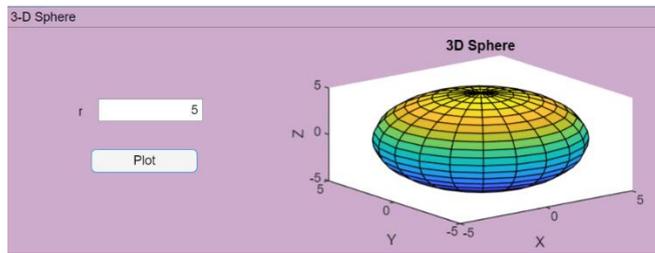


Fig. 12. The interface view of visualizing the sphere in three-dimensional space after execution

Fig. 12 illustrates the three-dimensional sphere for  $r = 5$ . The visualization confirms the ability of the application to accurately generate a three-dimensional sphere based on the provided radius,  $r$ .

## 5. CONCLUSION

In conclusion, an application named the Spherical First Order Polarization Tensor Calculator is developed using GUI MATLAB based on the developed algorithm. This study provides a quicker way to compute the first order PT for sphere, determine its radius based on its first order PT and visualize the sphere in three-dimensional space with just a single click. In addition, the reliability of this application is also provided through the demonstrations. This user-friendly application simplifies complex calculations, making it an efficient tool for researchers in related fields. For future enhancement, a new feature can be added to the application that focuses on determining the conductivity of the sphere from its first order PT, making the application more comprehensive for analyzing the first order PT of a sphere.

## 6. ACKNOWLEDGEMENTS/FUNDING

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## 7. CONFLICT OF INTEREST STATEMENT

The authors declare no conflict of interest.

## 8. AUTHORS' CONTRIBUTIONS

**Nur Izzati Abdul Rahman:** Analysis, software, writing - original draft, visualization; **Nurhazirah Mohamad Yunos:** Conceptualization, methodology, resources, writing – review and editing, supervision; **Taufiq Khairi Ahmad Khairuddin:** Conceptualization, methodology, resources.

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